Chapter 10

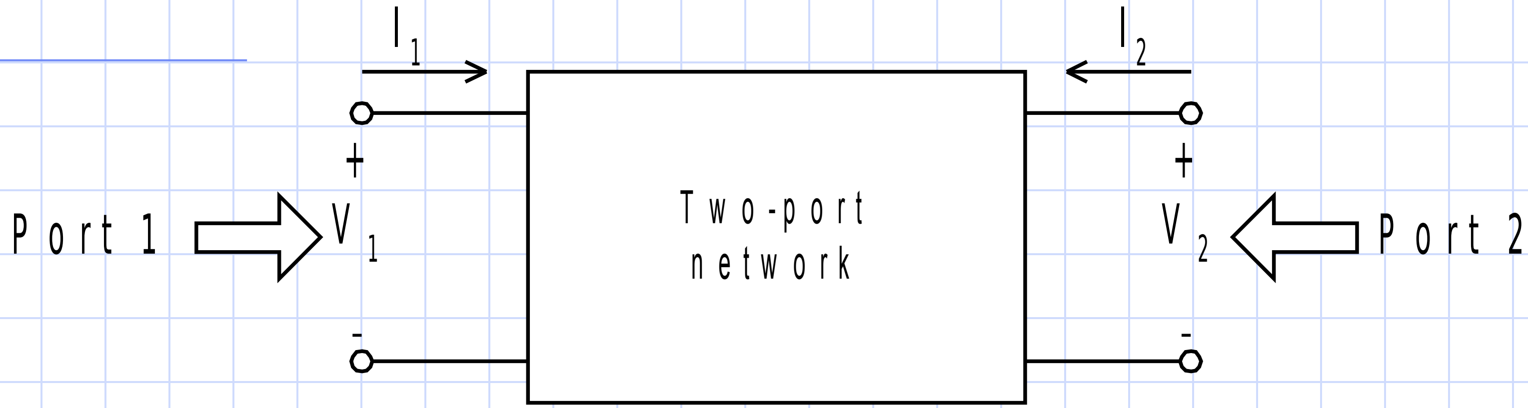
Two – Port Networks

EEE 33
1st Semester 2005-2006

Department of Electrical and Electronics Engineering

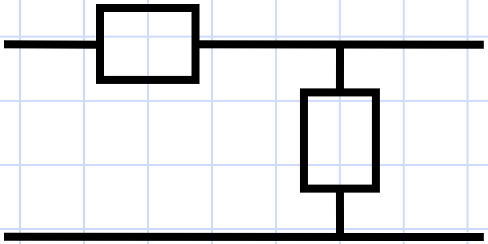
University of the Philippines - Diliman

Definition of Two-Port Networks

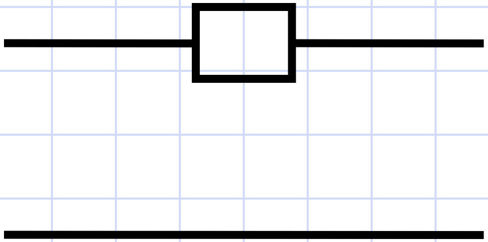


- In general, we describe a two-port network as a network consisting of R, L, and C elements, op-amps, transformers, and dependent sources, but no independent sources.
- Only two of the four variables are independent, and the specification of any two of them determines the remaining two.

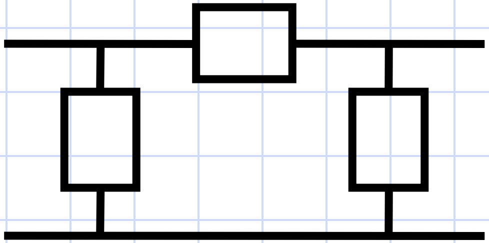
Examples of Two-Port Networks



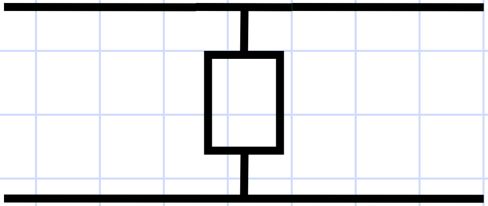
L - network



Series

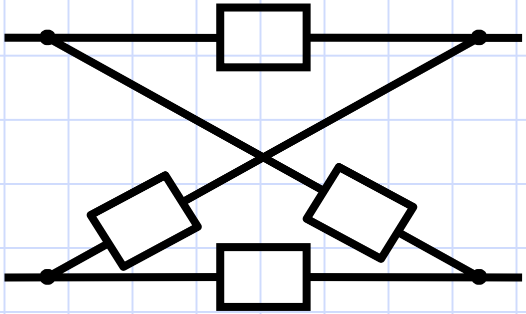


Pi network

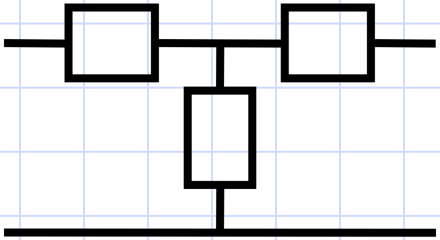


Shunt

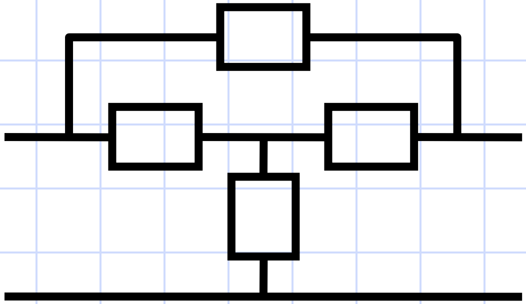
Examples of Two-Port Networks



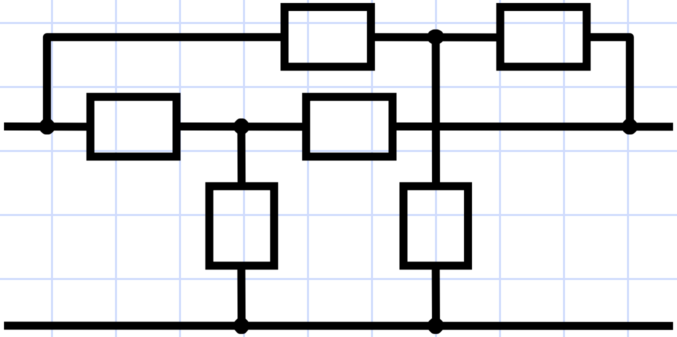
Lattice



T network

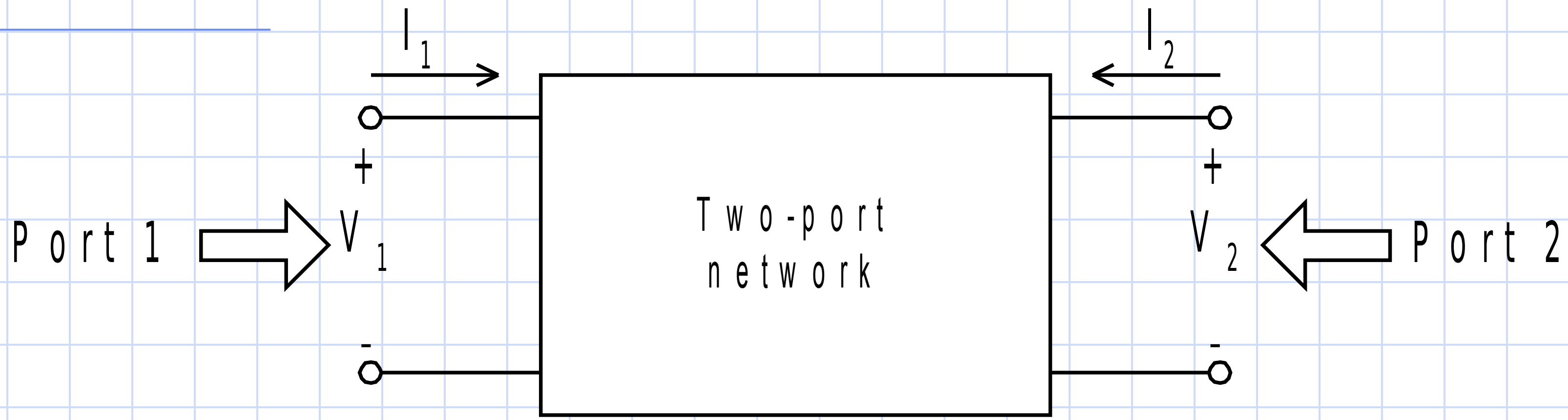


Bridged T



Twin T

Two-Port Network Parameters



Parameters of the two-port completely describe its behavior in terms of the voltage and current at each port.

- Permits us to describe its operation when it is connected to a larger network.
- Important in modeling electronic devices and system components.

Two-Port Parameters

- Admittance Parameters (y_{jk})
- Impedance Parameters (z_{jk})
- Hybrid Parameters (h_{jk})
- Transmission Parameters (t_{jk})

Admittance Parameters

- V_1 and V_2 are the independent variables
- Express I_1 and I_2 in terms of V_1 and V_2

$$I_1 = y_{11}V_1 + y_{12}V_2$$

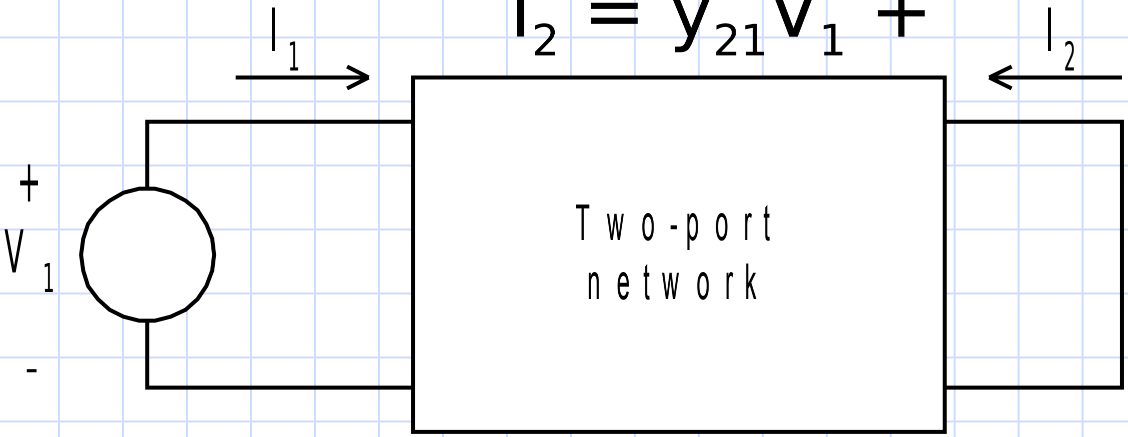
$$I_2 = y_{21}V_1 + y_{22}V_2$$

- The coefficients y_{jk} are also called the short-circuit admittance parameters or the y-parameters
- The coefficients y_{jk} are dimensionally admittance

Getting the Short-Circuit Admittance Parameters (y-parameters)

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 +$$



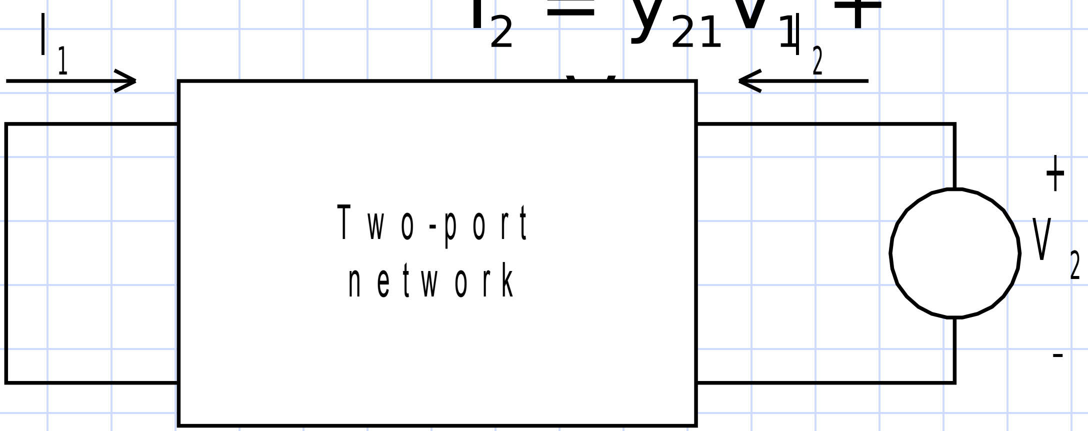
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Getting the Short-Circuit Admittance Parameters (y-parameters)

$$I_1 = y_{11}V_1 + y_{12}V_2$$

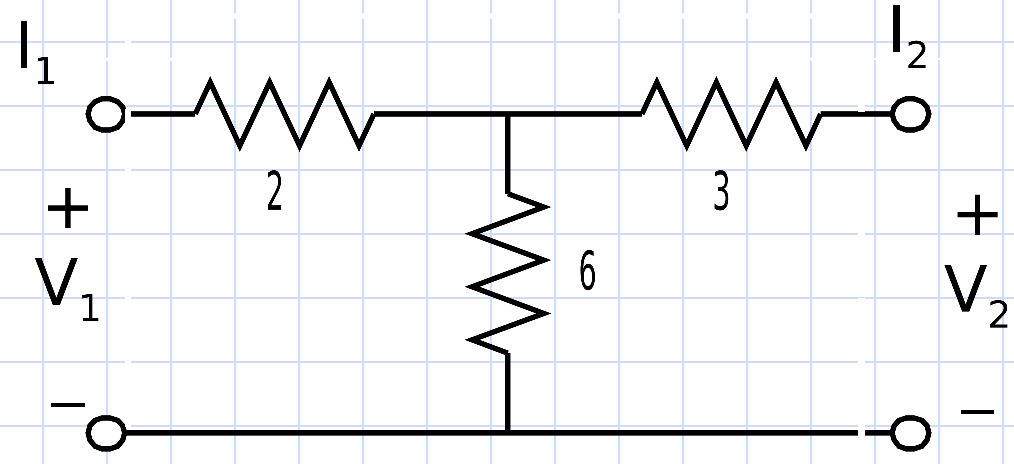
$$I_2 = y_{21}V_1 +$$



$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Example: Find the Y-parameters of the two-port network on the right.

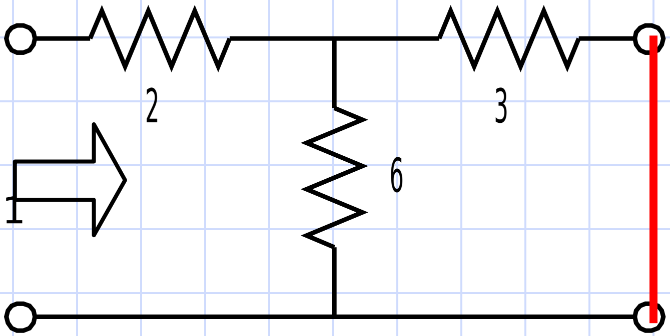


Get the y-parameters one by one.

y_{11}:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

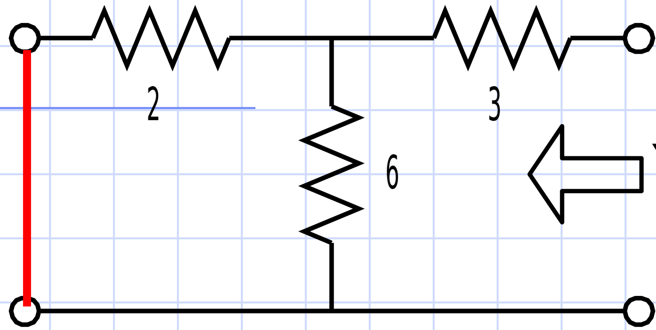
$Y_{in} = y_{11}$



$y_{11} = 1/(2 + 3//6)$ ℧

$= ¼$

y_{22}



$$Y_{in} = y_{22}$$

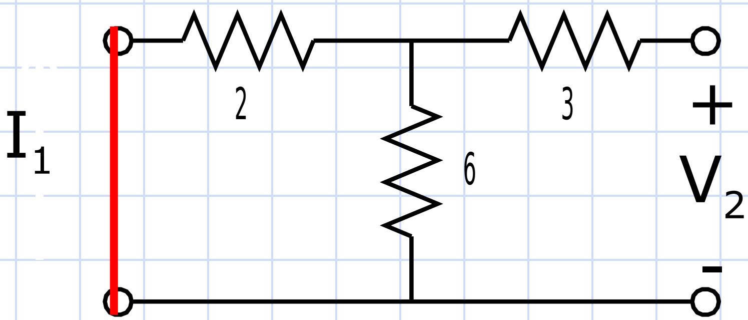
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y_{22} = 1/(3 + 2//6)$$

$$\mho$$

$$= 2/9$$

y_{12}

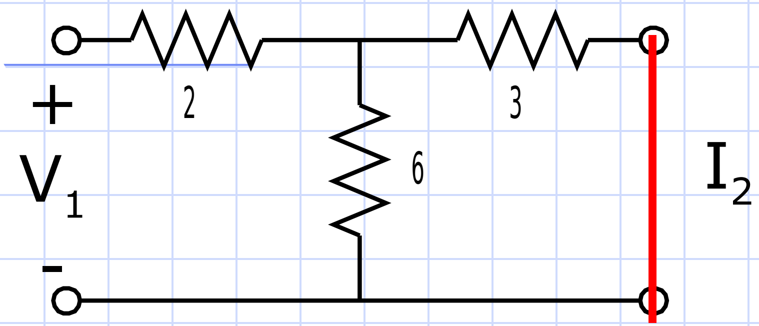


$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{12} = I_1 / V_2 = \frac{-\frac{3}{4} V_2 y_{22}}{V_2}$$

$$= -\frac{3}{4} y_{22} = -1/6 \ \mho$$

y_{21}

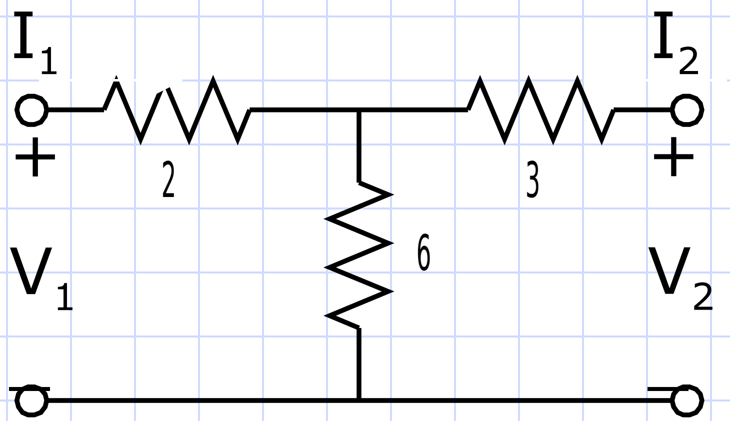


$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{21} = I_2 / V_1 = -\frac{2}{3} V_1 y_{11} / V_1$$

$$= -2/3 y_{11} = -1/6 \ \mho$$

Thus, we get



$$I_1 = 1/4\ V_1 - 1/6\ V_2$$
$$I_2 = -1/6\ V_1 + 2/9\ V_2$$

Or in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{2}{9} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Impedance Parameters

- I_1 and I_2 are the independent variables
- Express V_1 and V_2 in terms of I_1 and I_2

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

- Also known as the open-circuit impedance parameters or the z-parameters
- The coefficients are dimensionally impedance

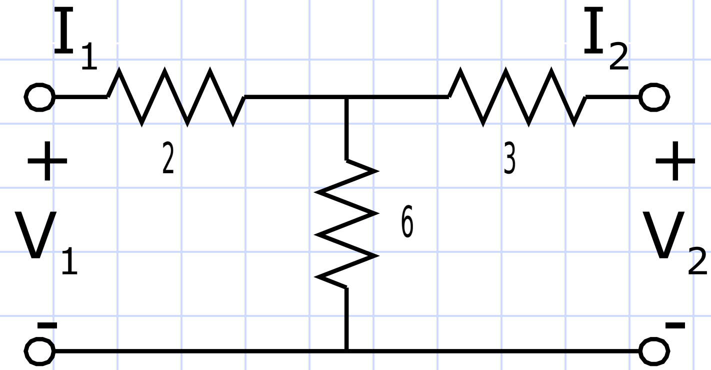
Impedance Parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \qquad z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \qquad z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

Example: Find the Z-parameters of the two-port network on the right.



Method 1: Using the definitions,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{11} = 2 + 6 = 8\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{21} = 6\ \Omega$$

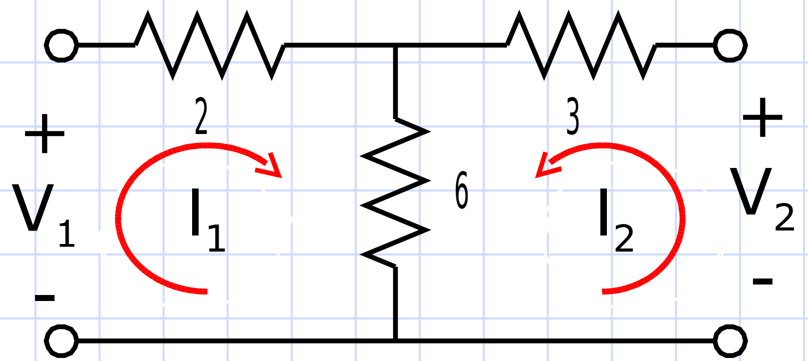
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{22} = 3 + 6 = 9\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$z_{22} = 6\ \Omega$$

Method 2: By writing loop equations,



$$V_1 = 2I_1 + 6(I_1 + I_2) = 8I_1 + 6I_2$$

$$V_2 = 3I_2 + 6(I_1 + I_2) = 6I_1 + 9I_2$$

Therefore,

$z_{11} = 8$, $z_{12} = 6$, $z_{21} = 6$ and $z_{22} = 9$

Relationship Between Y- and Z-parameters

Let $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ and $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

We know $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ $\quad I = YV$

Solving for V: $\mathbf{V = Y^{-1}I}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad V = ZI$$

Therefore,

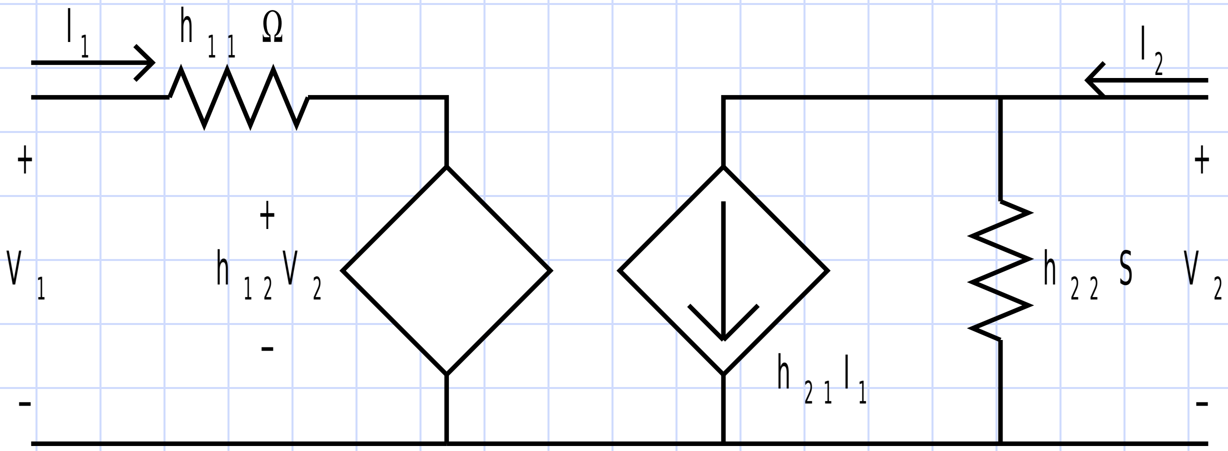
$$Z = Y^{-1} \text{ or } \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^{-1}$$

Hybrid Parameters

- Also known as H-parameters
- Often used to model the small signal behavior of bipolar junction transistors

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$



Getting the H-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \qquad h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} \qquad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

Converting Z- to H-parameters

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad V_2 = z_{21}I_1 + z_{22}I_2$$

Express V_1 and I_2 with in terms of V_2 and I_1

$$I_2 = -\frac{z_{21}}{z_{22}}I_1 + \frac{1}{z_{22}}V_2$$

$$V_1 = z_{11}I_1 + z_{12}\left(-\frac{z_{21}}{z_{22}}I_1 + \frac{1}{z_{22}}V_2\right)$$

$$= \left(z_{11} - z_{12}\frac{z_{21}}{z_{22}}\right)I_1 + \frac{z_{12}}{z_{22}}V_2$$

$$h_{11} = z_{11} - \frac{z_{12}z_{21}}{z_{22}}$$

$$h_{12} = \frac{z_{12}}{z_{22}}$$

$$h_{21} = -\frac{z_{21}}{z_{22}}$$

$$h_{22} = \frac{1}{z_{22}}$$

Inverse Hybrid Parameters

- Also known as G-parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

Transmission Parameters

- The transmission parameters relate the voltage and current at one port of the network with those of the other end.

$$V_1 = t_{11}V_2 - t_{12}I_2 \qquad\qquad V_1 = AV_2 - BI_2$$
$$\text{or}$$
$$I_1 = t_{21}V_2 - t_{22}I_2 \qquad\qquad I_1 = CV_2 - DI_2$$

- The transmission parameters are also known as the chain parameters, general circuit parameters, or ABCD parameters

Transmission Parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad -\frac{1}{B} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad \frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad -\frac{1}{D} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

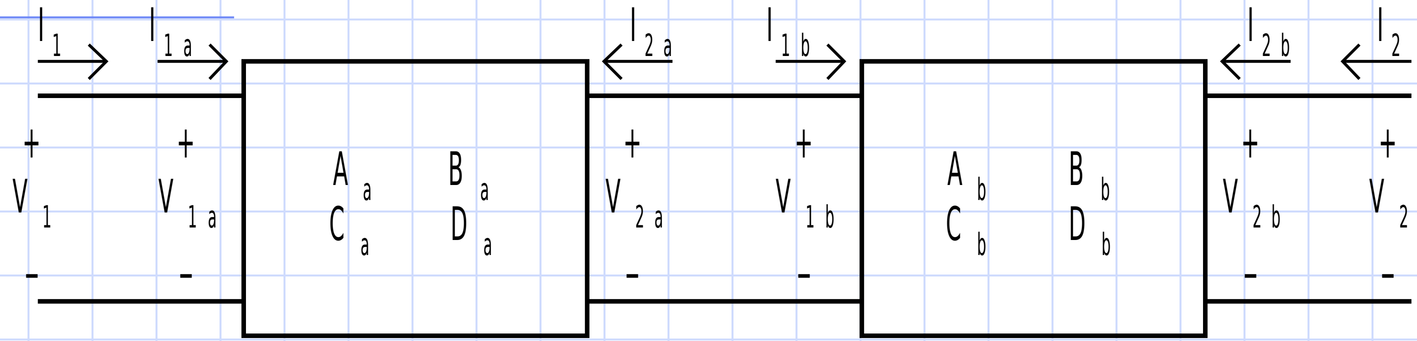
1/A - open-circuit voltage gain

-1/B - short-circuit transfer admittance

1/C - open-circuit transfer impedance

-1/D - short-circuit current gain

The t-parameters are useful in describing two-port networks which are connected in cascade.



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\text{but} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} \text{ so } \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Therefore, in

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

Inverse Transmission Parameters

- Also known as ABCD′ parameters

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$

- For passive networks,

$$AD - BC = A'D' - B'C' = 1$$