Linear Circuits
Part 1

EEE 31

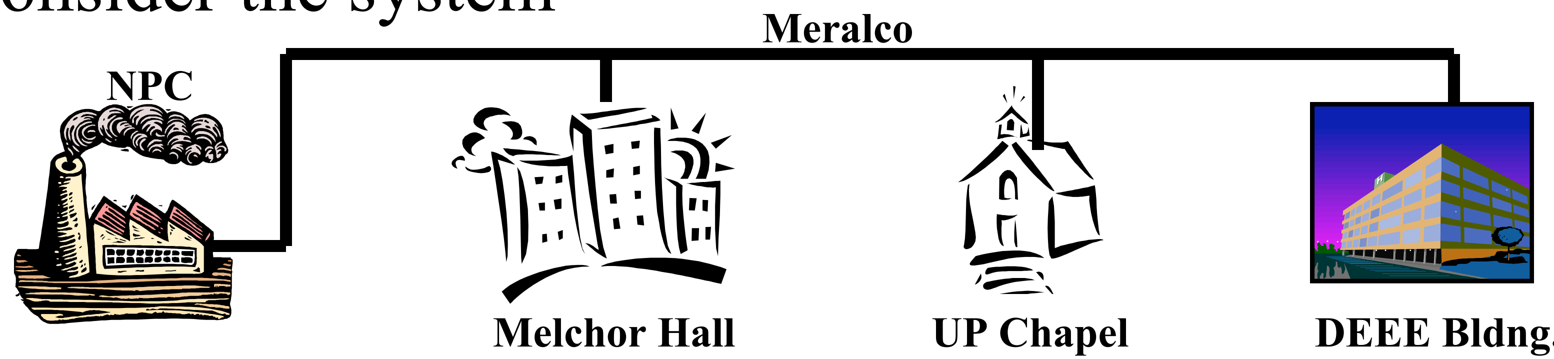
Introduction to Electrical and Electronics Engineering

Lecture Outline

- Lumped and Distributed Models
- Network/Circuit
- Conservation of Charge
- KCL
- Conservation of Energy
- KVL

Lumped and Distributed Modeling

- Consider the system

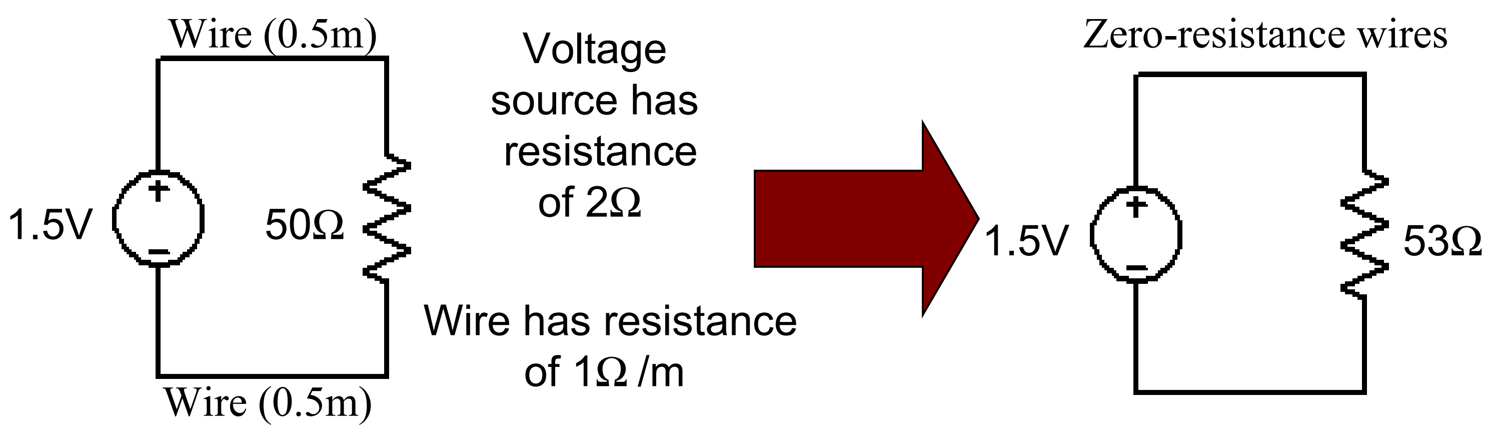


  - Meralco views the total consumption of an entire household as **load**
  - The only information you get is the KWH you consumed when you get an electric bill

- Consider also an **electric fan**
  - motor ➔ wires, stator, rotor ➔ resistance per line

Lumped and Distributed Modeling

- Distributed Parameter Modeling
  - modeling of an electrical circuit into an infinite number of vanishingly small elements
- Lumped Parameter Modeling
  - modeling of an electrical circuit into a number of simple elements

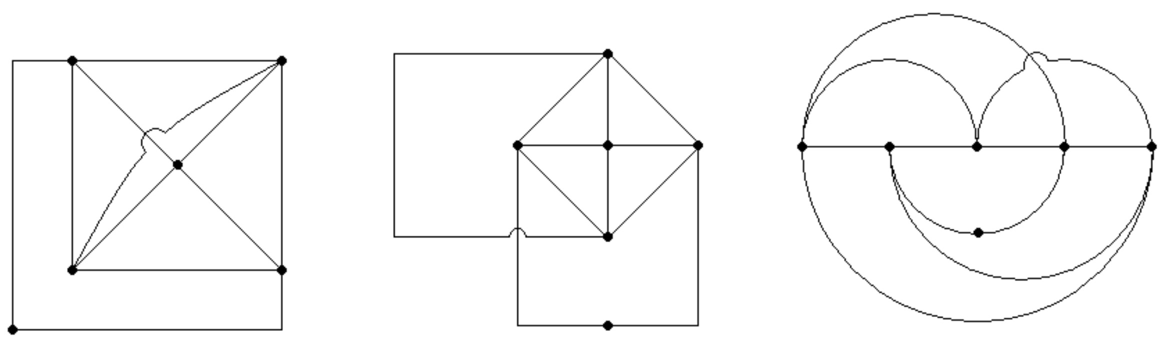


Network / Circuit

- **Current** is measured as the flow of charge **through** the cross section of a circuit element
- **Voltage** is measured **across** the ends of a circuit element
- **Electrical circuit** consist of various types of *circuit elements* connected by *conductors*
  - Circuit elements: sources, resistors, inductors and capacitors
  - Conductors are wires connecting circuit elements

Network / Circuit

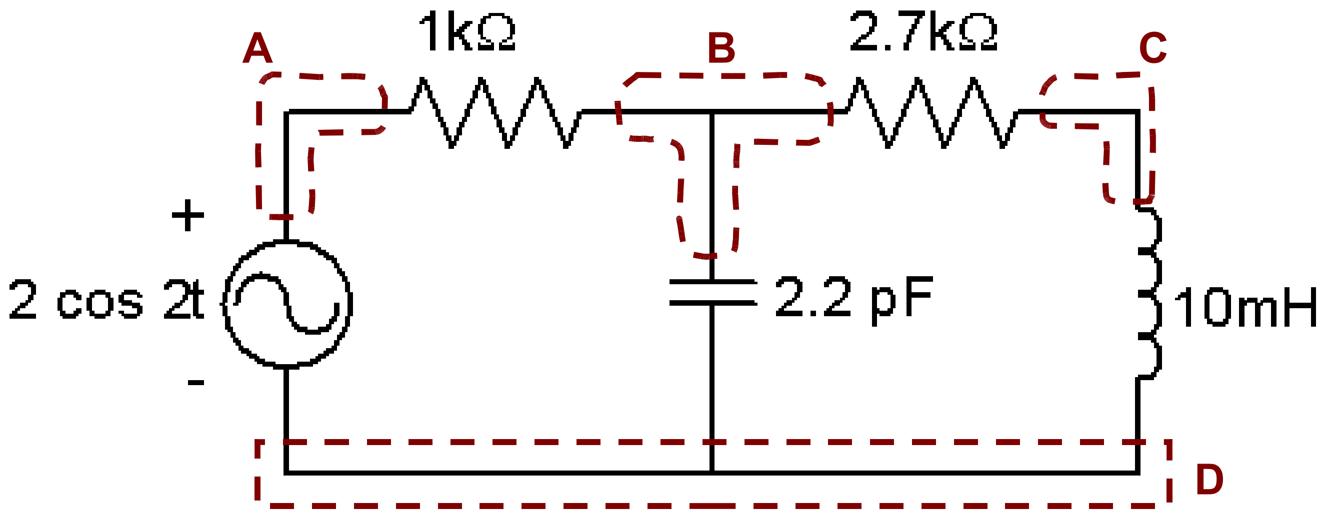
- **Topology**
  - Branch of geometry which is concerned with those properties of a geometrical figure which are **unchanged** when the figure is twisted, bent, folded, stretched, squeezed, or tied in knots, with the provision that **NO parts of the figure are to be cut apart or to be joined together**



Figures of the same topology.

Network / Circuit

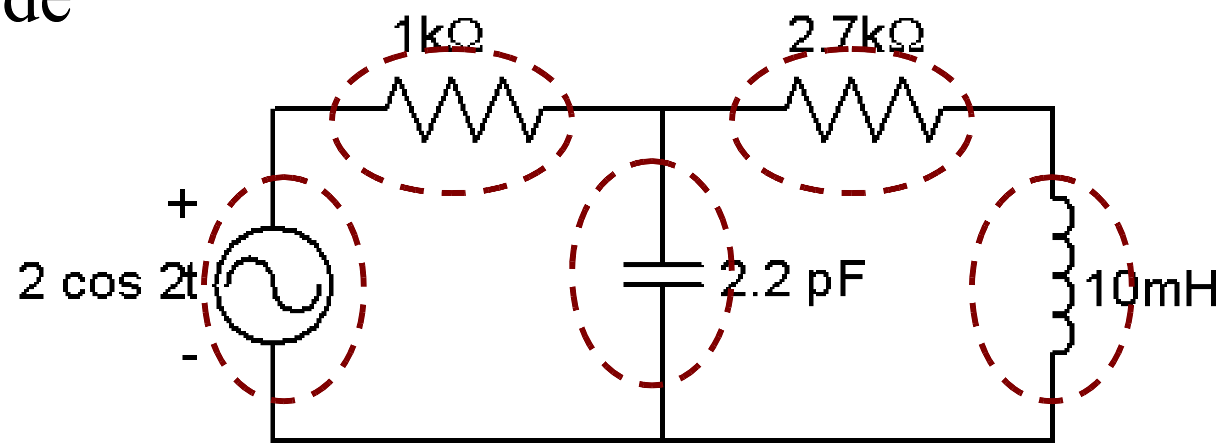
- **Nodes** - where two or more circuit elements are connected



5 circuit elements
4 nodes

Network / Circuit

- **Path** – a set of elements that may be traversed in order without passing through the same node twice
- **Branch** – a single path, containing one simple element, which connects one node to any other node

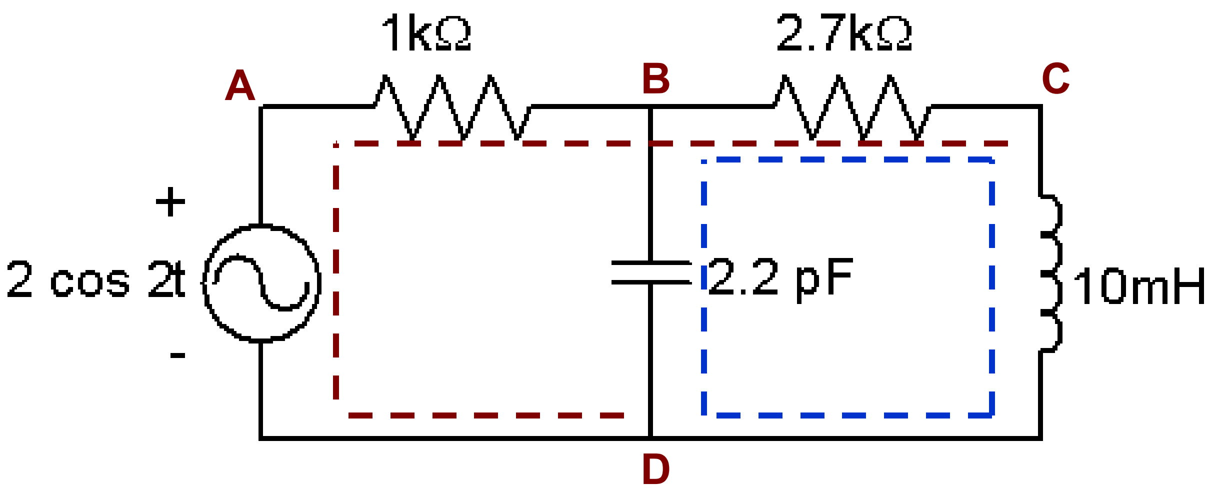


5 circuit elements

5 branches

Network / Circuit

- **Path**

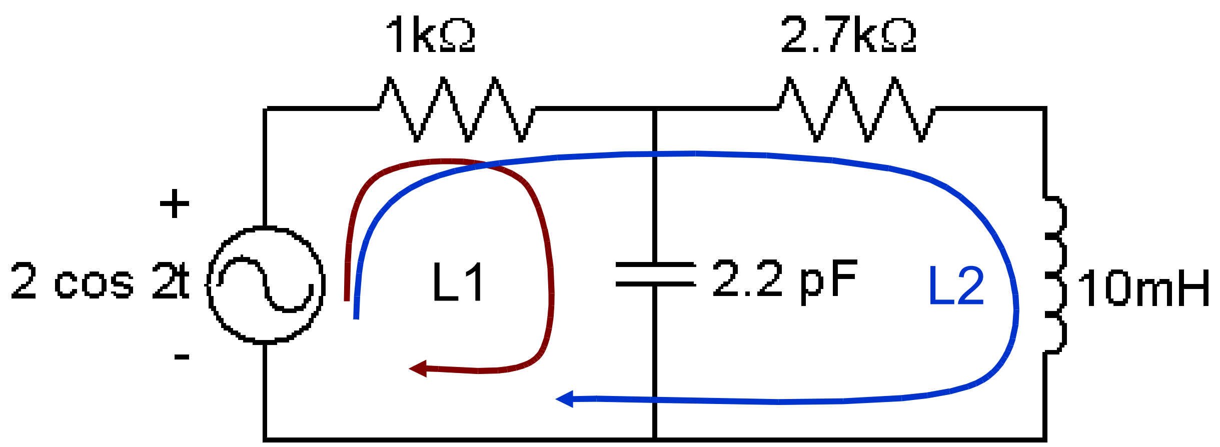


Path 1: going through source, 1kΩ and 2.7kΩ resistors

Path 2: going through 2.2pF, 2.7kΩ and 10mH

Network / Circuit

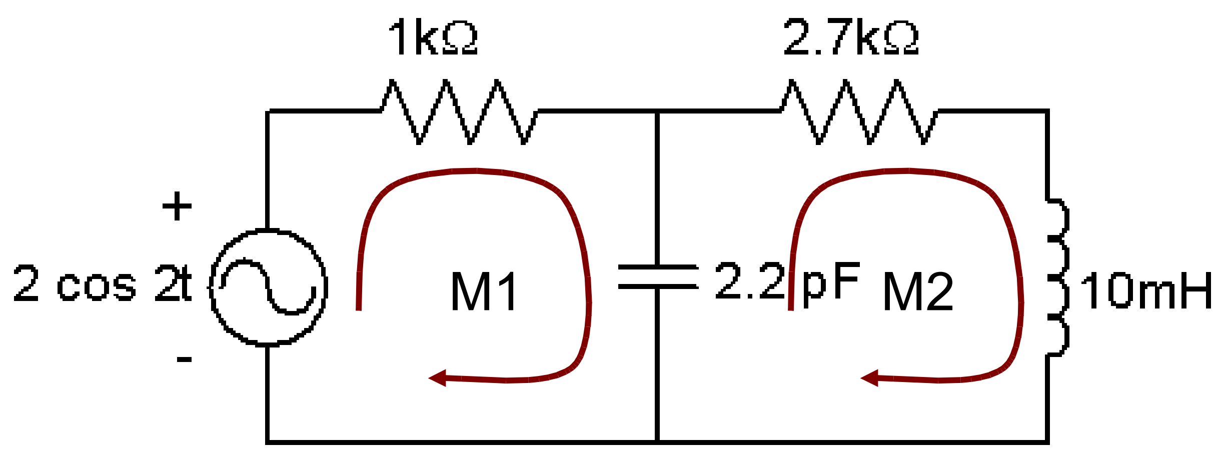
- **Loop** – a closed path
  - In an electrical circuit, a loop is a closed path starting at a node and proceeding through circuit elements, eventually returning to the starting node.



L1 and L2 are loops.

Network / Circuit

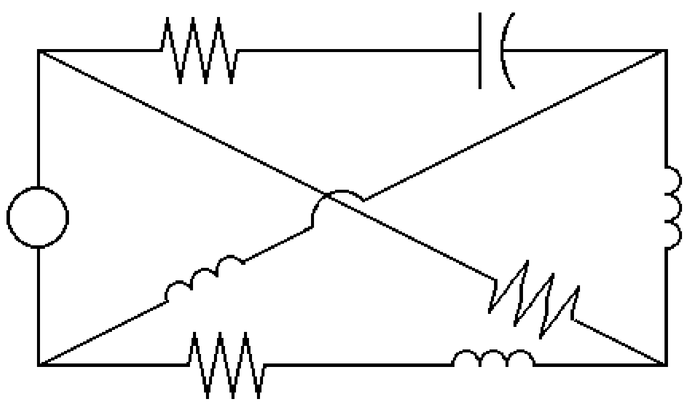
- **Mesh** – a loop that does not contain any other loops within it

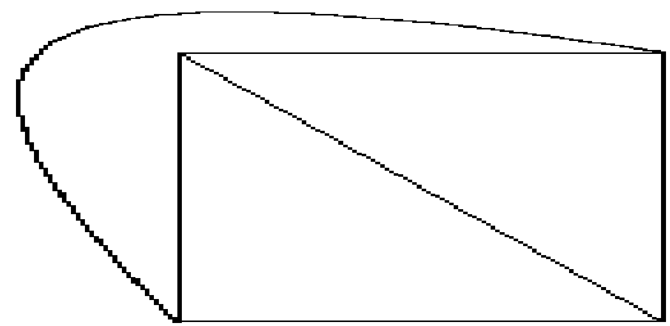


M1 and M2 are meshes.

Network / Circuit

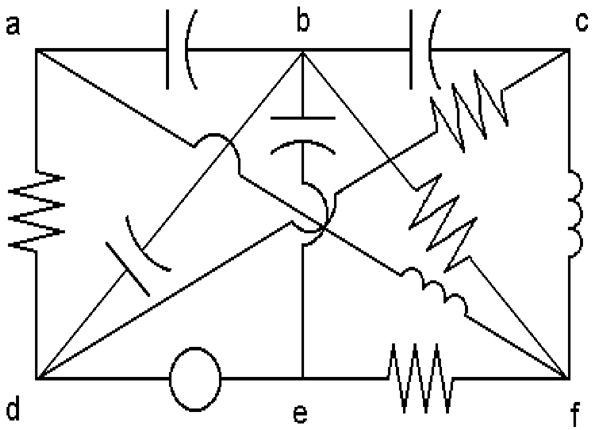
- **Planar circuits -**a circuit which may be drawn on a plane surface in such a way that no branch passes over or under any other branch

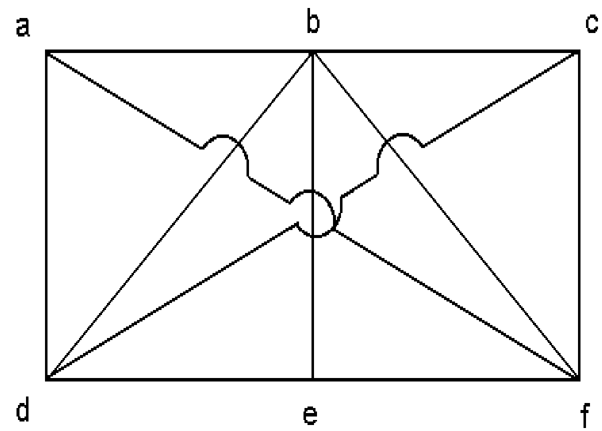




Network / Circuit

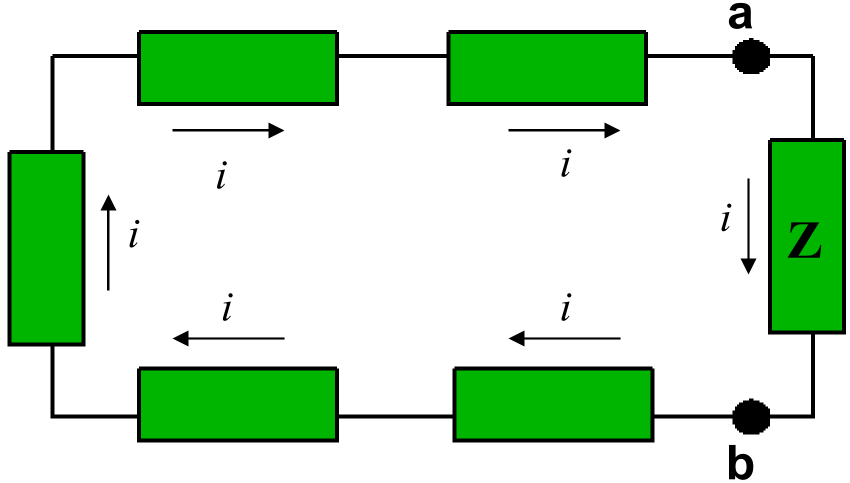
- **Non-planar circuits -** any circuit that is not planar.

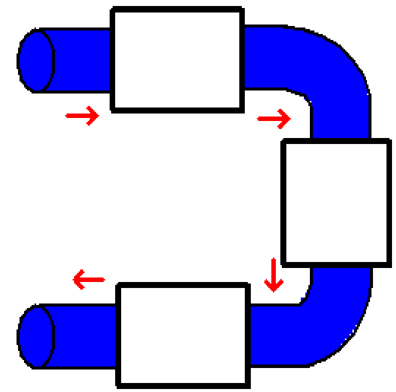




Series Connection

- Consider the circuit



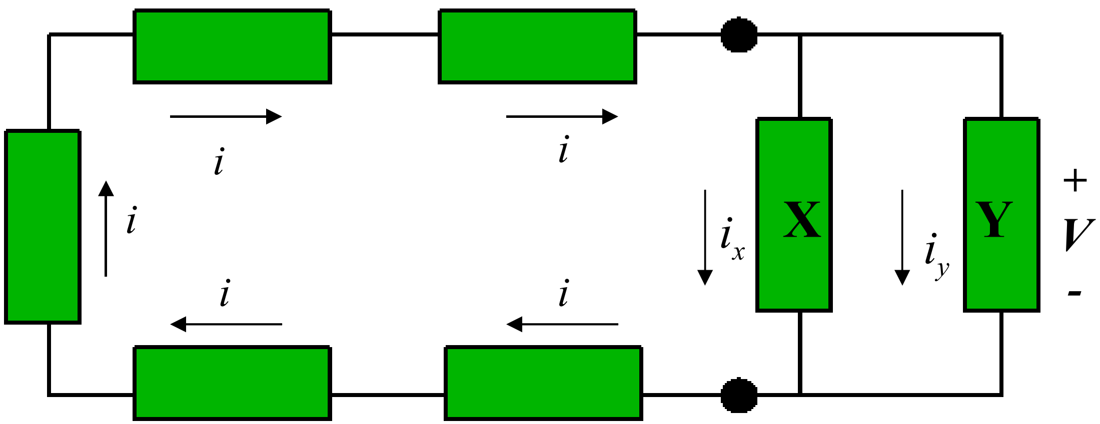


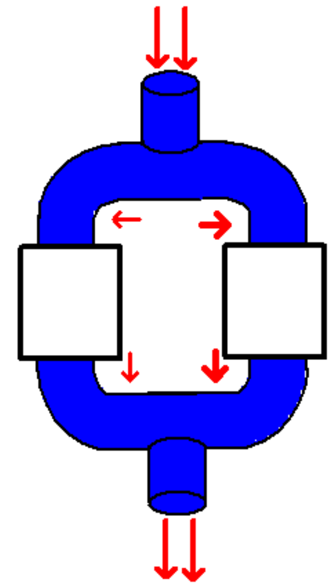
Water flow analogy

- The circuit comprises a loop.
- Every node has only 2 elements connected to it.
- The **current through** each element is the **same**.

Parallel Connection

- Consider the circuit





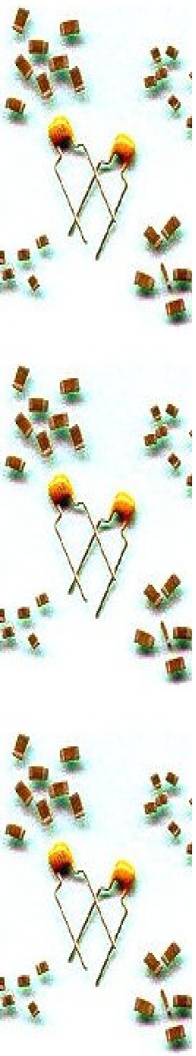
- Same circuit as before except for an extra element.
- The **voltage across** the elements X & Y is the **same**.

Conservation of Charge

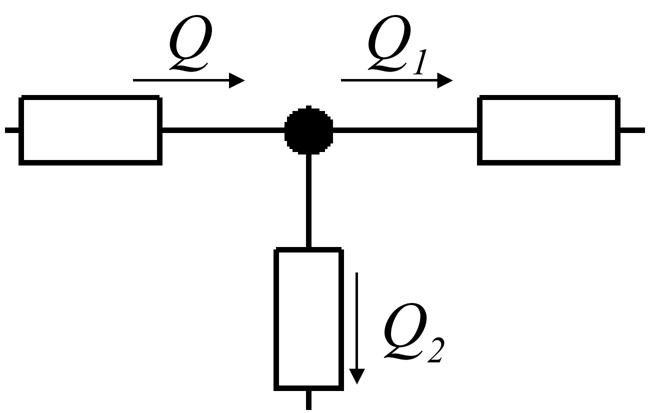
The algebraic sum of all electric charges in any closed system is constant.

Charge can be transferred from one body to another, but it cannot be created or destroyed.

Conservation of Charge



- Consider the junction in the circuit:



- **Conservation of Charge**

$$Q = Q_1 + Q_2$$

Taking the derivative

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta Q_1}{\Delta t} + \frac{\Delta Q_2}{\Delta t}$$

But $i = \frac{\Delta Q}{\Delta t}$

$$i = i_1 + i_2$$

Kirchhoff's Current Law

The algebraic sum of the currents entering a node equals zero.

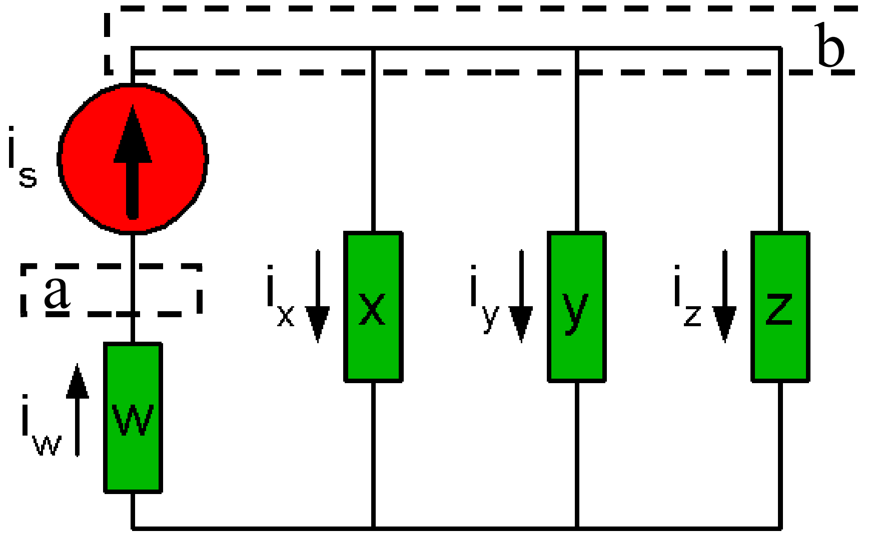
- KCL is a re-statement of conservation of charge
- Convention:

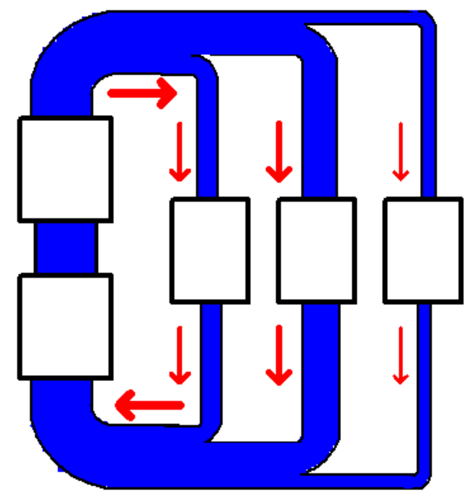
  '+' for currents entering a node

  '-' for currents leaving a node

Total current entering a node = Total current leaving a node

Example: KCL





(a) $i_w = i_s$

(b) $i_s - i_x - i_y - i_z = 0$ or $i_s = i_x + i_y + i_z$

Conservation of Energy

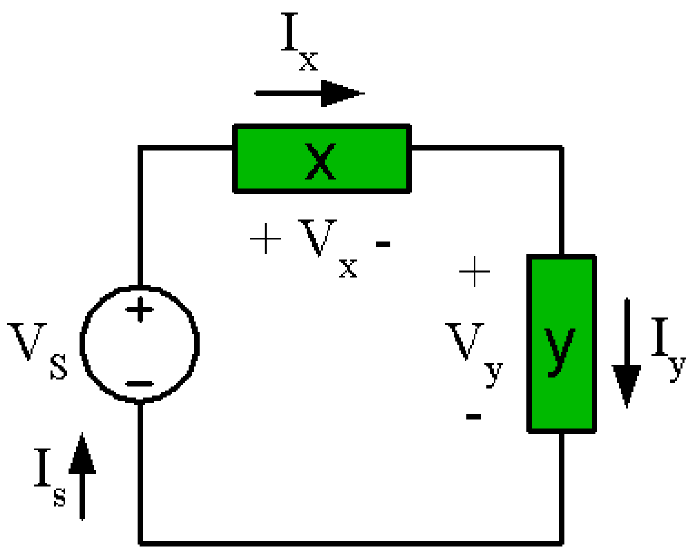
Energy can not be created or destroyed,

it is converted

from one form to another.

Conservation of Energy

- Consider the circuit:



- Dissipated energy for each element

  - Voltage Source

$$P_s = -V_s I_s$$

  - Element *x*

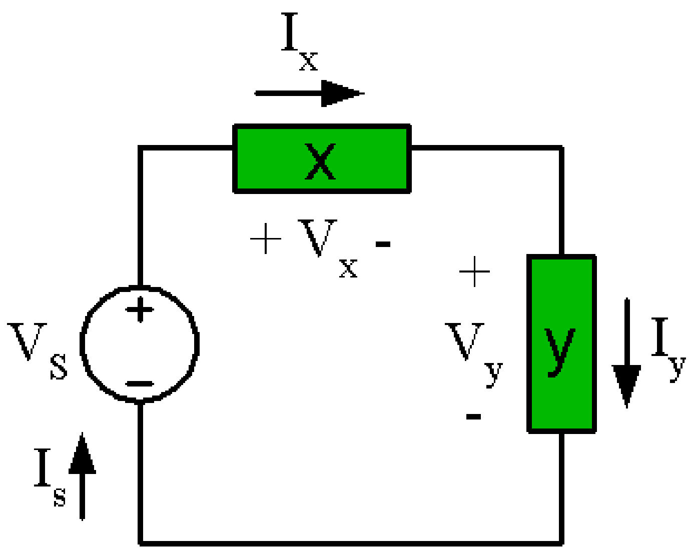
$$P_x = V_x I_x$$

  - Element *y*

$$P_y = V_y I_y$$

Conservation of Energy

- Consider the circuit:



- At any instant,

$$\sum P = 0$$

$$P_s + P_x + P_y = 0$$

$$-V_s I_s + V_x I_x + V_y I_y = 0$$

But $I_s = I_x = I_y$

$$-V_s I_s + V_x I_s + V_y I_s = 0$$

Canceling I_s

$$-V_s + V_x + V_y = 0$$

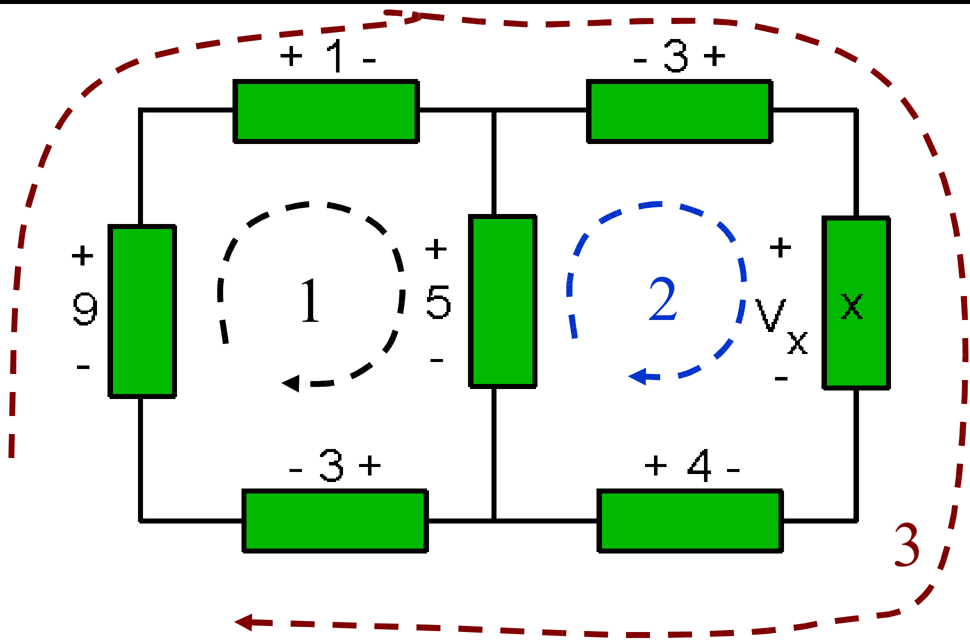
Kirchhoff's Voltage Law

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit.

- KVL is a re-statement of conservation of energy
- Convention: Use the first polarity mark encountered at each voltage in the algebraic sum

$$\text{Total voltage rise} = \text{Total voltage drop}$$

Example: KVL

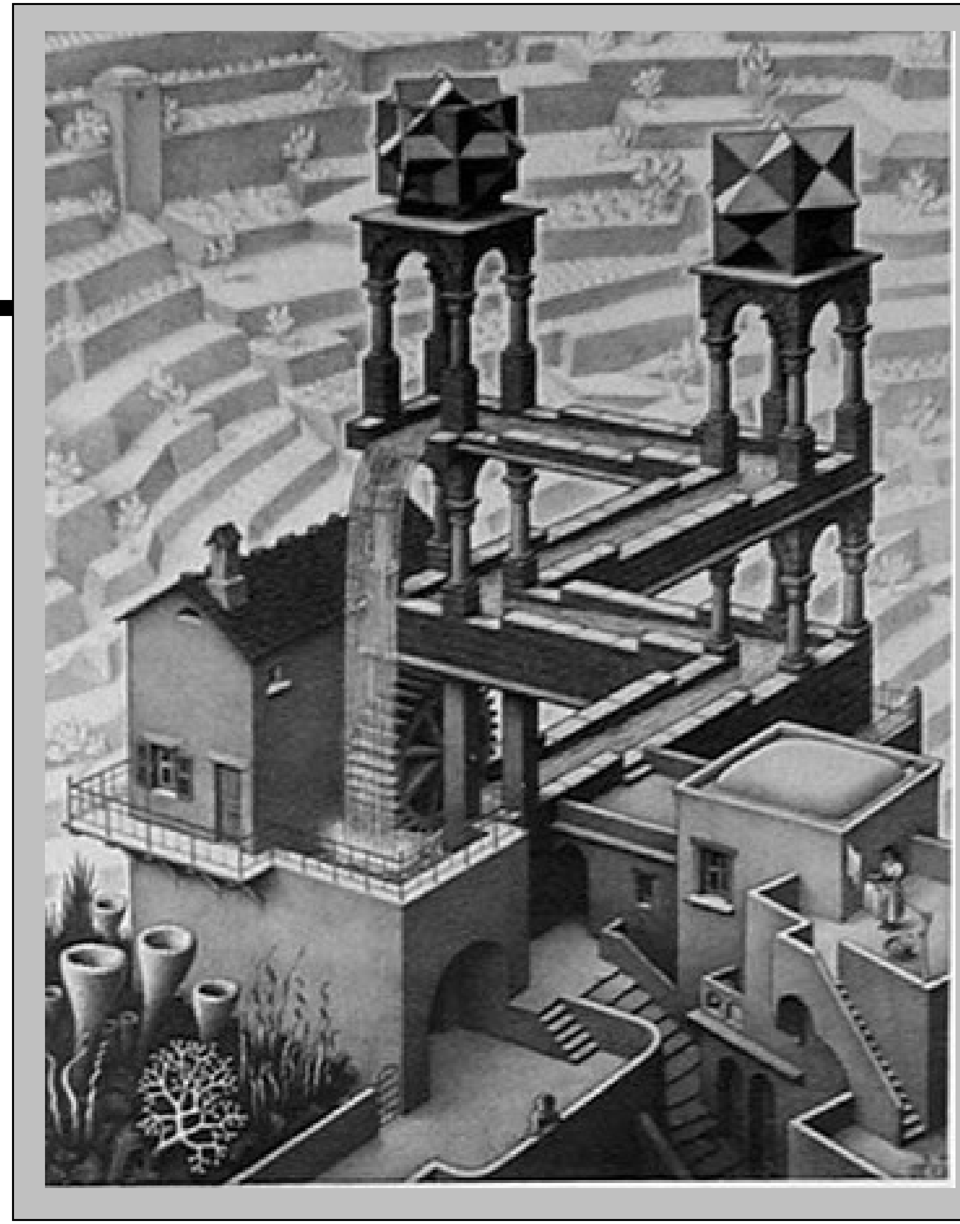


Loop 1: $-9 + 1 + 5 + 3 = 0$

Loop 2: $-3 + V_X - 4 - 5 = 0$ or $V_X = 12$

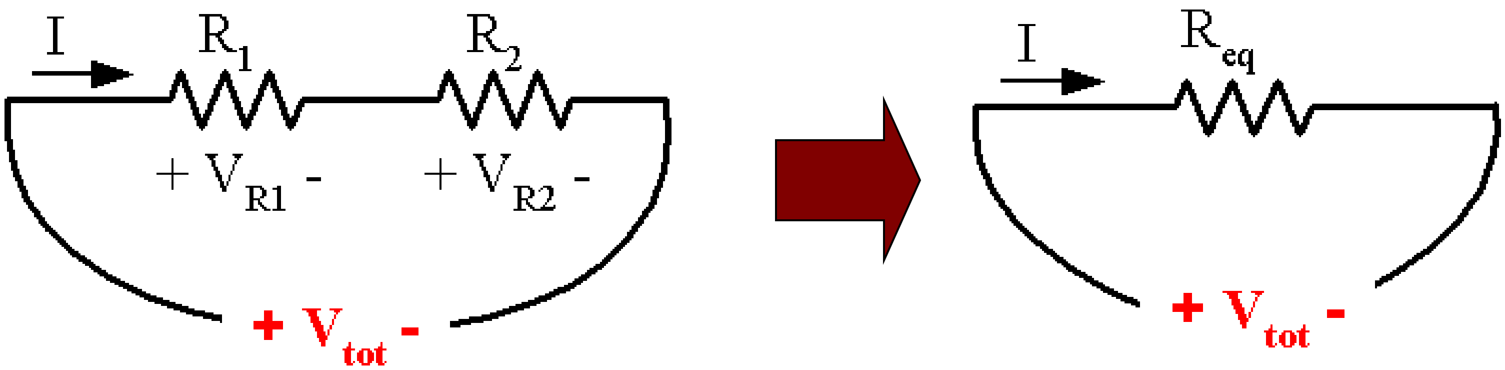
Loop 3: $-9 + 1 - 3 + 12 - 4 + 3 = 0$

KVL



“Waterfall”
By M.C. Escher
http://www.worldofescher.com/gallery/A63L.html

Resistors in Series

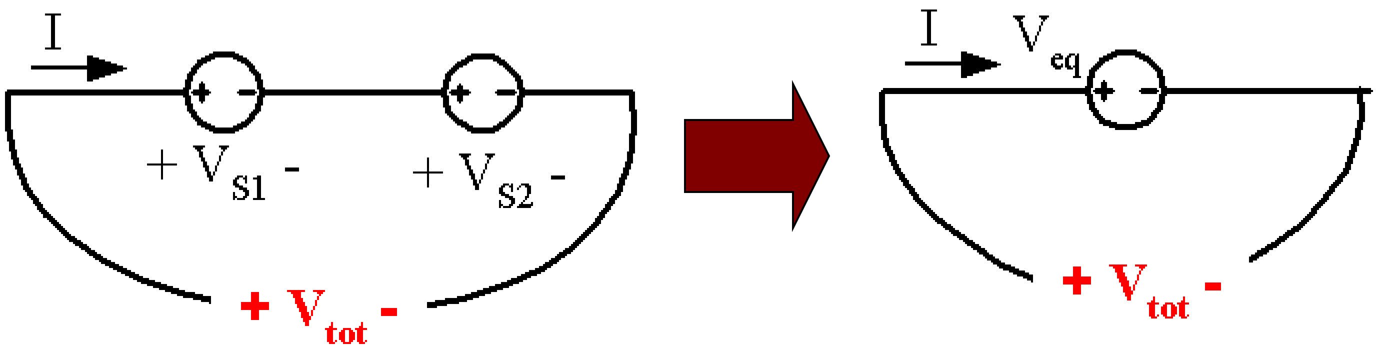


KVL: $V_{tot} = V_{R1} + V_{R2}$

Ohm's Law: $IR_{eq} = IR_1 + IR_2$

Equivalent Resistance: $R_{eq} = R_1 + R_2$

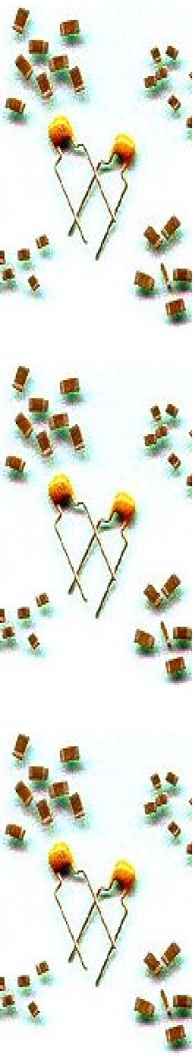
Voltage Source in Series

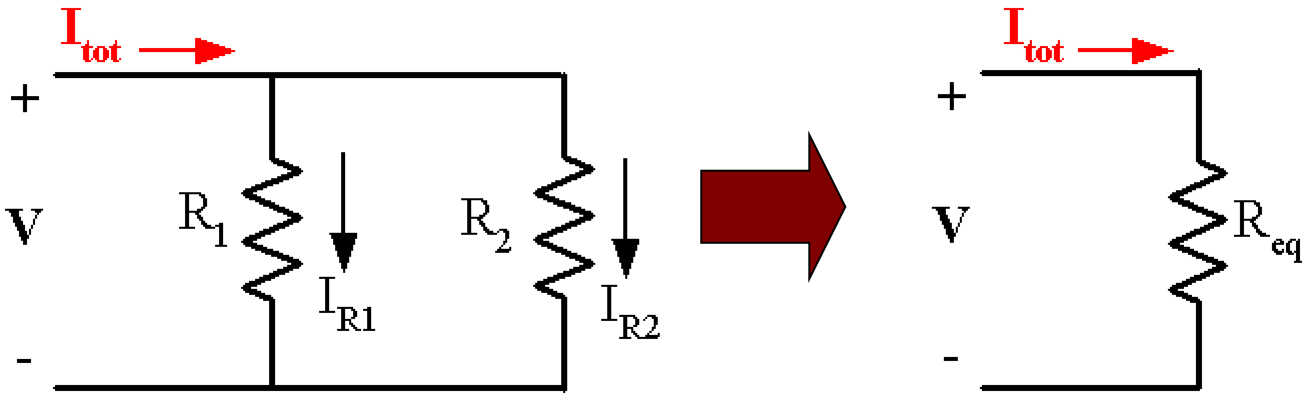


Equivalent Voltage: $V_{eq} = V_{S1} + V_{S2}$

You cannot put two voltage sources in parallel unless they have the same value.

Resistors in Parallel





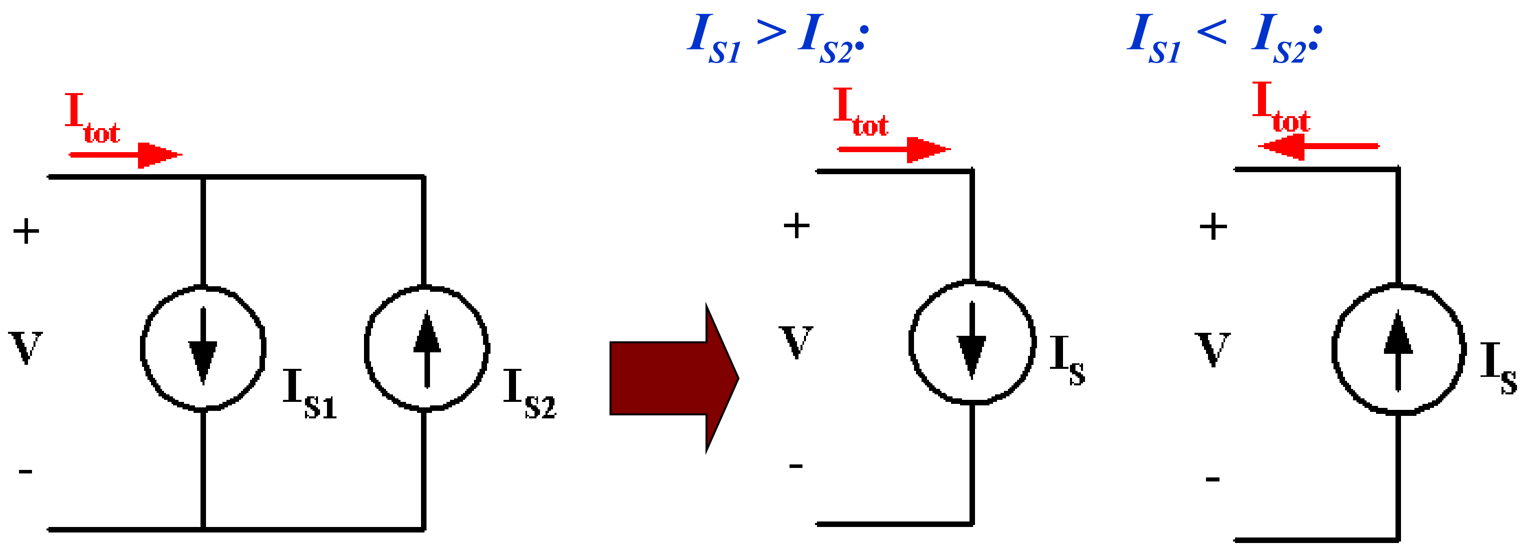
KCL: $$I_{tot} = I_{R1} + I_{R2}$$

Ohm's Law: $$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

Two Resistors in Parallel: $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Equivalent Resistance: $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Current Sources in Parallel

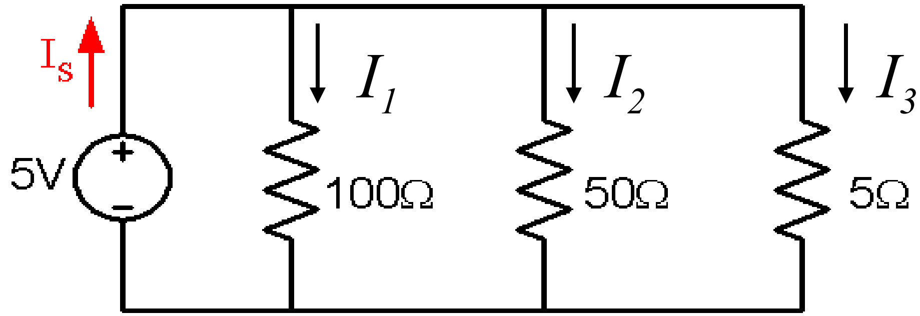


Equivalent Current: $I_{tot} = I_{S1} - I_{S2}$

You cannot put two current sources in series unless they have same value.

Example 1

Find the current I_S.



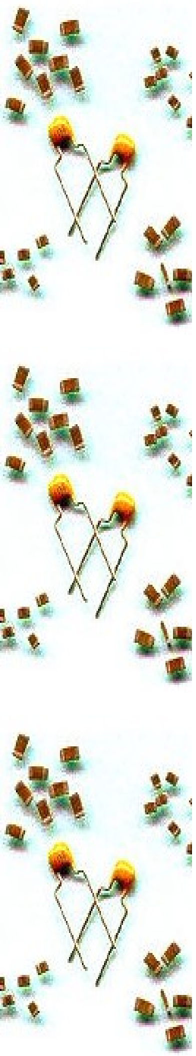
KCL: $I_S = I_1 + I_2 + I_3$

KVL: $I_1 = \dfrac{5V}{100\,\Omega} = 50\,mA$ $\quad I_2 = \dfrac{5V}{50\,\Omega} = 100\,mA$ $\quad I_3 = \dfrac{5V}{5\,\Omega} = 1A$

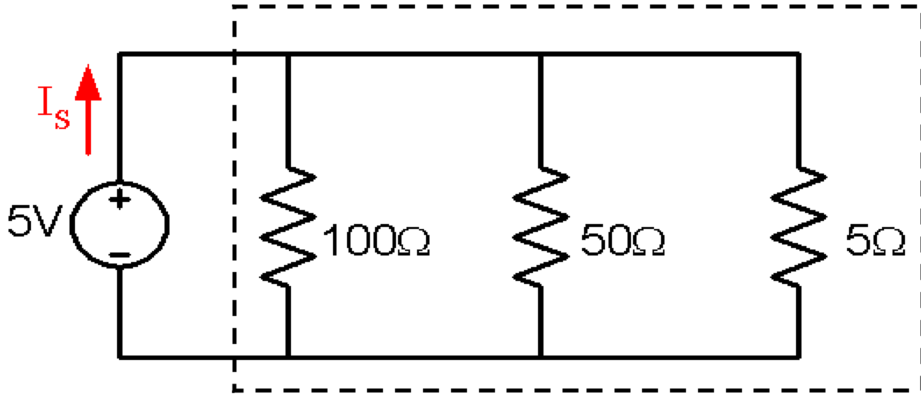
Plugging in the values,

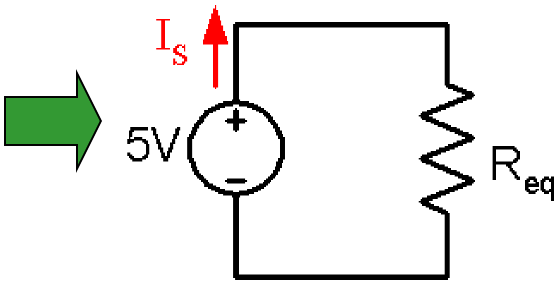
$$I_S = 50\text{mA} + 100\text{mA} + 1\text{A} = \mathbf{1.15A}$$

Example 1 : Alternative Solution



Simplify the resistors.





Determine R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{100\,\Omega} + \frac{1}{50\,\Omega} + \frac{1}{5\,\Omega}$$

$$\frac{1}{R_{eq}} = 0.23 \quad \text{or } R_{eq} = 4.348\Omega$$

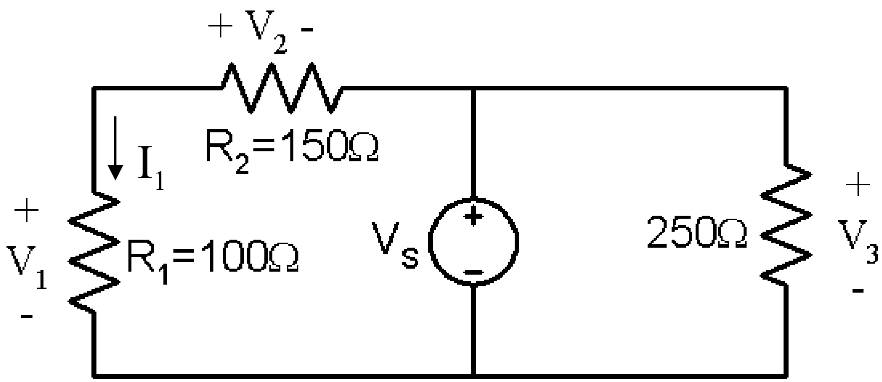
KVL:

$$I_s = \frac{5\text{V}}{R_{eq}} = \frac{5\text{V}}{4.348\,\Omega}$$

$$I_S = \mathbf{1.15A}$$

Resistors in parallel: If $R_1 < R_2 < R_3$ …, then $\mathbf{R_{eq} < R_1}$

Example 3



Find the voltages V_S, V_2 and V_3 given $V_1 = 4\text{V}$.

Ohm's Law: $I_1 = \dfrac{V_1}{100\,\Omega} = 40\text{ mA}$

Since R_1 and R_2 are in series, I_1 is the same current through R_2

Ohm's Law: $V_2 = -I_1 R_2$

$= -0.04(150)$

$V_2 = -6\text{V}$

KVL around left loop:

$-V_1 + V_2 + V_s = 0$

$V_s = V_1 - V_2$

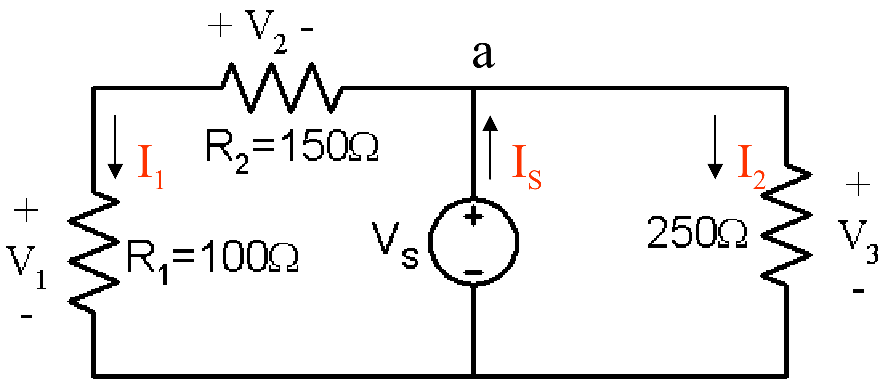
$= 4 - (-6)$

$V_s = 10\text{V}$

V_S and V_2 are in parallel,

$V_3 = V_s = 10\text{V}$

Example 4



Find the current I_S.

KCL at node a: $I_s = I_1 + I_2$

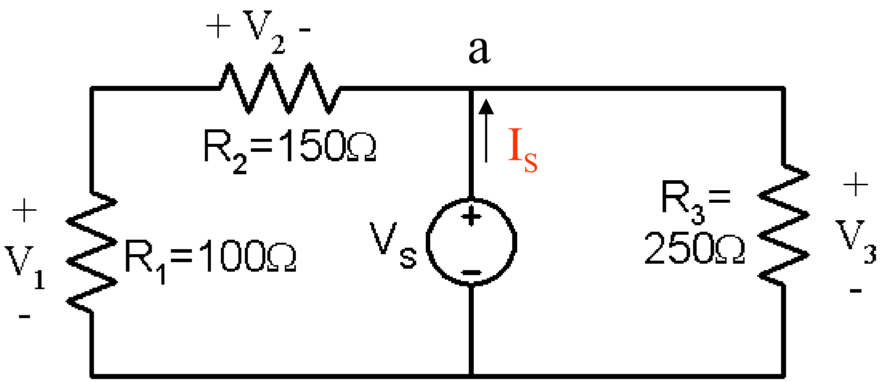
From previous example, we found I_1= 40 mA and V_3= 10V

Ohm's Law: $I_2 = \dfrac{V_3}{250\,\Omega} = 40\text{ mA}$

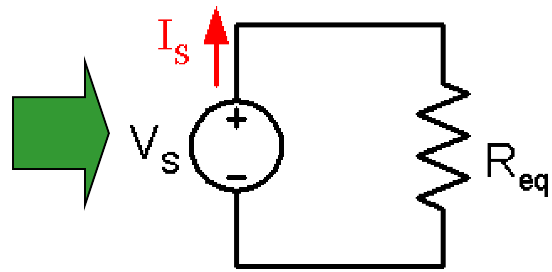
Plugging in to the first equation,

$$I_s = 40\text{ mA} + 40\text{ mA} = 80\text{ mA}$$

Example 4 : Alternative Solution



Get the equivalent resistance

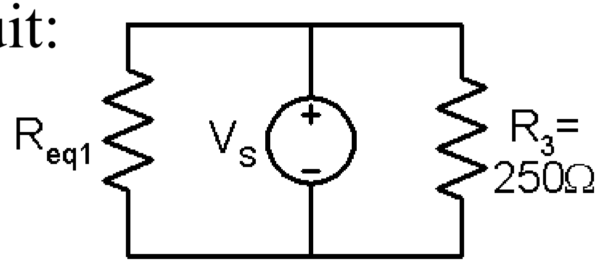


From previous example, we found $V_S = 10\text{V}$

R_1 in series with R_2 : $R_{eq1} = R_1 + R_2$

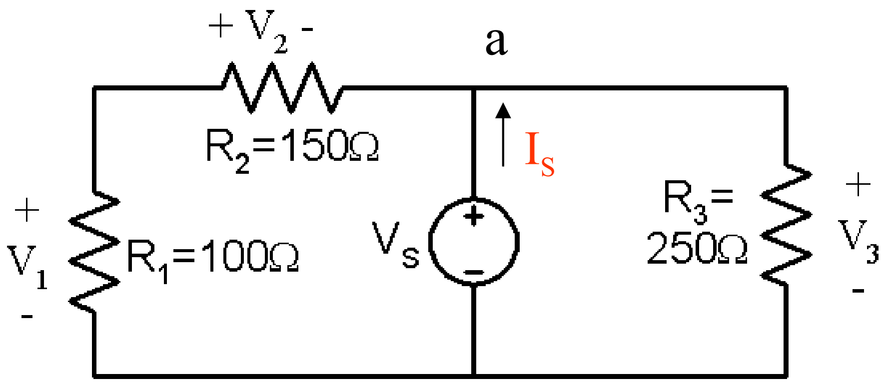
$$R_{eq1} = 100\Omega + 150\Omega = 250\Omega$$

Equivalent circuit:

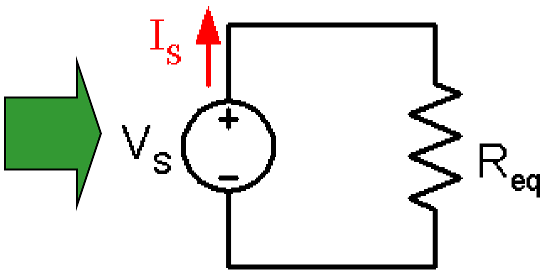


R_{eq1} is in parallel with R_3

Example 4 : Alternative Solution



Get the equivalent resistance



R_{eq1} is in parallel with R_3 :

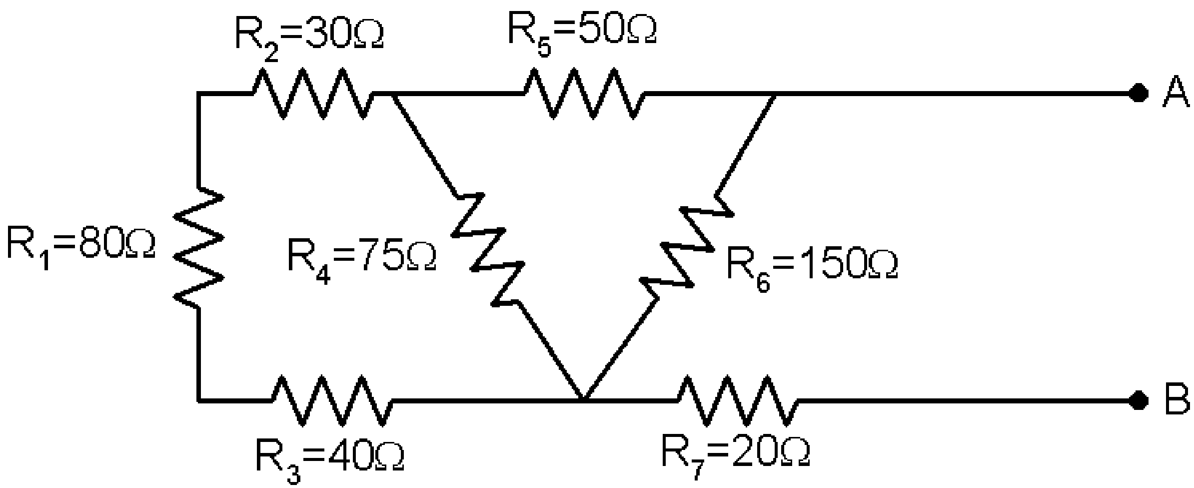
$$R_{eq} = \frac{R_{eq\,1}\left(R_3\right)}{R_{eq\,1}+R_3}$$

$$= \frac{250\left(250\right)}{250+250} = 125\,\Omega$$

The resistance of two equal resistors in parallel is equivalent to half the original resistance.

Ohm's Law: $I_S = \dfrac{V_S}{R_{eq}} = \dfrac{10}{125} = 80\,mA$

Example 5



Find the equivalent resistance between nodes A and B.

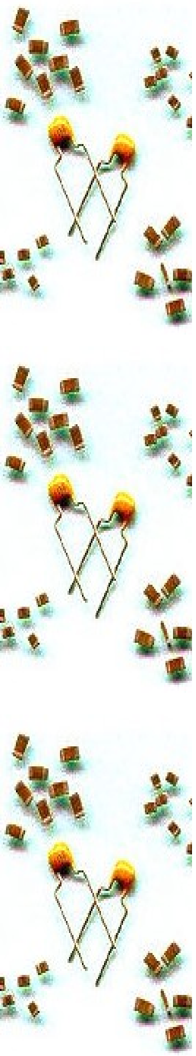
Simplify the circuit from left to right.

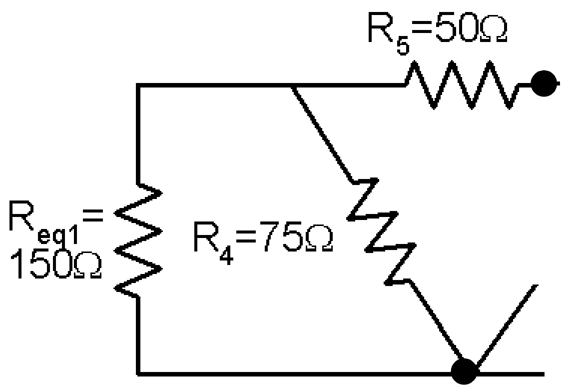
Series resistance: $R_{eq1} = R_1 + R_2 + R_3$

$$= 80\Omega + 30\Omega + 40\Omega$$

$$R_{eq1} = 150\Omega$$

Example 5



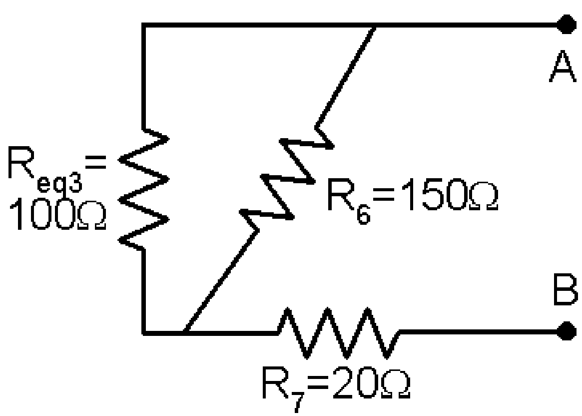


R_{eq1} parallel R_4 :

$$R_{eq2} = \frac{R_{eq1}(R_4)}{R_{eq1} + R_4} = \frac{150(75)}{150+75}$$

$$R_{eq2} = 50\Omega$$

R_{eq2} series R_5 : $\quad R_{eq3} = 50 + 50 = 100\Omega$



R_{eq3} parallel R_6 :

$$R_{eq3} = \frac{100(150)}{100+150} = 60\ \Omega$$

R_{eq3} series R_7 :

$$\boldsymbol{R_{AB}} = 60 + 20 = 80\Omega$$