EEE 31 Problem Set 2 Part I

1 Part 1 Prob 1

1.1 Using source transformations

Left-side of variable Resistance (R)

$$\begin{aligned} I_{s,left} &= \frac{15}{300} \\ R_{s,left} &= 300||150 \\ &= \frac{(300)(150)}{300+150} \\ &= 100\Omega \\ V_{s,left} &= \frac{(15)(100)}{300} \\ &= 5V \end{aligned}$$

Right-side of variable Resistance (R)

$$\begin{aligned} I_{s,right} &= \frac{40}{120} \\ R_{s,right} &= 120||480 \\ &= \frac{(120)(480)}{120+480} \\ &= 96\Omega \end{aligned}$$

$$V_{s,right} = \frac{(40)(96)}{120}$$

$$= 32V$$

$$\begin{aligned} V_{Th} &= 32-5 \\ &= 27V \\ R_{Th} &= 100+96 \\ &= 196\Omega \end{aligned}$$

$$P_{max} = \frac{(\frac{27}{2})^2}{196}$$

1.2 Using Mesh Current Method

Thevenin Voltage (higher potential at the right terminal)

$$\begin{aligned} V_{Th} &= 40\frac{480}{480+120} - 15\frac{150}{150+300} \\ &= 32-5 \\ &= 27V \end{aligned}$$

Thevenin Resistance

$$\begin{aligned} R_{Th} &= (300||150)+(480||120) \\ &= 196\Omega \end{aligned}$$

2 Part 1 Prob 2

To get the Thevenin equivalent, remove R_L, then analyze across that terminal-pair

2.1 Using Superposition

With voltage source acting alone

$$\begin{aligned} i_x &= 10i_x \\ i_x &= 0 \\ V'_{OC} &= 3(10i_x) \\ V'_{OC} &= 0 \\ I'_{SC} &= 10i_x \\ &= 0A \end{aligned}$$

With current source acting alone

$$\begin{aligned} i_x + 0.9 &= 10i_x \\ i_x &= 0.1A \\ V''_{OC} &= 3(10i_x) \\ V''_{OC} &= 3V \\ I''_{SC} &= 10i_x \\ &= 1A \end{aligned}$$

$$\begin{aligned} V_{OC} &= 0 + 3 \\ &= 3V \\ I_{SC} &= 0 + 1 \\ &= 1A \\ R_{Th} &= 3\Omega \end{aligned}$$

2.2 Using KCL

$$\begin{aligned} i_x + 0.9 &= 10i_x \\ i_x &= 0.1A \\ V_{OC} &= 3(10i_x) \\ &= 3V \\ I_{SC} &= 10i_x \\ &= 1A \\ R_{Th} &= 3\Omega \end{aligned}$$

For maximum power transfer:

$$\begin{aligned} R_L &= R_{Th} \\ &= 3\Omega \\ P_{max} &= \frac{(\frac{3}{2})^2}{3} \\ &= 0.75W \end{aligned}$$

3 Part 1 Prob 3

3.1 Calculating I_0

Remove center $1k\Omega$ resistor, then get the open circuit voltage (V_{OC}), higher potential at the left terminal. Using Node Voltage Method with bottom node as reference, top node as V_1, center left node as V_2 and center right node as V_3.

$$
\begin{aligned}
\frac{V_1 - 12}{2k} + \frac{V_1 - V_2}{1k} + \frac{V_1 - 4}{2k} &= 2m \\
\frac{V_2 - V_1}{1k} &= 2I_x \\
I_x &= \frac{V_3}{1k} \\
2m + \frac{V_3}{1k} &= 0 \\
V_3 &= -2V \\
V_2 &= 2V \\
V_{OC} &= V_2 - V_3 \\
V_{OC} &= 4V
\end{aligned}
$$

Replace center $1k\Omega$ resistor with short circuit, then get the Norton current by calculating the short circuit current (I_{SC}) to the right. Using Node Voltage Method with bottom node as reference, top node as V_1, and center node as V_2.

$$
\begin{aligned}
\frac{V_1 - 12}{2k} + \frac{V_1 - V_2}{1k} + \frac{V_1 - 4}{2k} &= 2m \\
\frac{V_2 - V_1}{1k} + \frac{V_2}{1k} + 2m &= 2I_x \\
I_x &= \frac{V_2}{1k} \\
V_1 &= 2V \\
V_2 &= -4V \\
I_{SC} &= 2m + \frac{V_2}{1k} \\
I_N &= -4mA
\end{aligned}
$$

$$
\begin{aligned}
R_N &= \frac{V_{OC}}{I_{SC}} \\
&= -1k\Omega
\end{aligned}
$$

3.2 Calculating V_0

Remove right $2k\Omega$ resistor, then get the Thevenin voltage as open circuit voltage (V_{OC}), higher potential at top terminal. Using Node Voltage Method with bottom node as reference, top node as V_1, center left node as V_2 and center right node as V_3.

$$
\begin{aligned}
\frac{V_1 - 12}{2k} + \frac{V_1 - V_2}{1k} &= 2m \\
\frac{V_2 - V_1}{1k} + \frac{V_2 - V_3}{1k} &= 2I_x \\
I_x &= \frac{V_3}{1k} \\
\frac{V_3 - V_2}{1k} + \frac{V_3}{1k} &= -2m \\
V_1 &= -4V \\
V_{OC} &= V_1 - 4 \\
V_{Th} &= -8V
\end{aligned}
$$

Replace right $2k\Omega$ resistor with short circuit, then calculate the short circuit current downwards. Using Node Voltage Method with bottom node as reference, top node as V_1, center left node as V_2 and center right node as V_3.

$$\begin{aligned}
V_1 = 4 & \\
\frac{V_2 - 4}{1k} + \frac{V_2 - V_3}{1k} &= 2I_x \\
I_x &= \frac{V_3}{1k} \\
2m + \frac{V_3}{1k} + \frac{V_3 - V_2}{1k} &= 0 \\
V_2 &= 2V \\
V_3 &= 0V \\
I_{SC} &= 2m + \frac{V_2 - V_1}{1k} + \frac{12 - V_1}{2k} \\
I_N &= 4mA
\end{aligned}$$

$$\begin{aligned}
R_{Th} &= \frac{V_{OC}}{I_{SC}} \\
&= -2k\Omega
\end{aligned}$$

4 Part 1 Prob 4

$$\begin{aligned}
V_{out1} &= -V_1 \\
V_{out2} &= \frac{V_2}{2} \\
V_{out3} &= 2V_3 \\
V_o &= \frac{-120k}{40k}V_{out1} + \frac{-120k}{120k}V_{out2} + \frac{-120k}{30k}V_{out3} \\
V_o &= 3V_1 - 0.5V_2 - 8V_3
\end{aligned}$$

5 Part 1 Prob 5

$$\begin{aligned}
V_a &= -12V \\
V_b &= -12\left(\frac{20k}{40k + 20k}\right) \\
&= -4V \\
V_c &= V_b \\
&= -4V \\
V_d &= V_c \\
V_d &= -4V \\
\frac{V_d - V_e}{20k} + \frac{V_d - V_f}{20k} &= 0 \\
\frac{V_g - V_f}{20k} + \frac{V_g}{40k} &= 0 \\
V_g &= V_e \\
V_e &= -\frac{16}{5}V \\
V_f &= -\frac{24}{5}V
\end{aligned}$$