PROBLEMS

B-3-1. Simplify the block diagram shown in Figure 3–50 and obtain the closed-loop transfer function $C(s)/R(s)$.

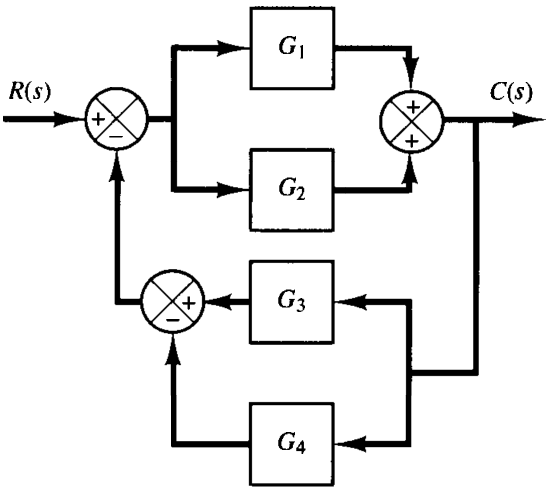

Figure 3–50 Block diagram of a system.

B-3-2. Simplify the block diagram shown in Figure 3–51 and obtain the transfer function $C(s)/R(s)$.

B-3-3. Simplify the block diagram shown in Figure 3–52 and obtain the closed-loop transfer function $C(s)/R(s)$.

B-3-4. Obtain a state-space representation of the system shown in Figure 3–53.

B-3-5. Consider the system described by

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Derive a state-space representation of the system.

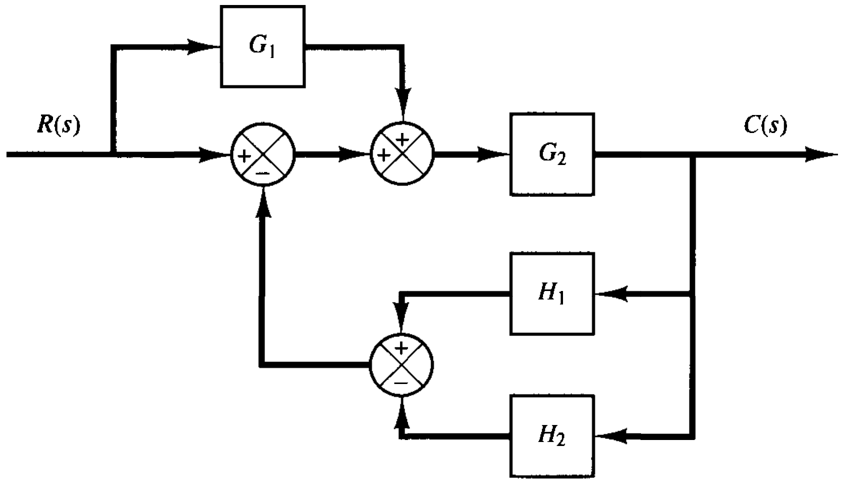

Figure 3–51 Block diagram of a system.

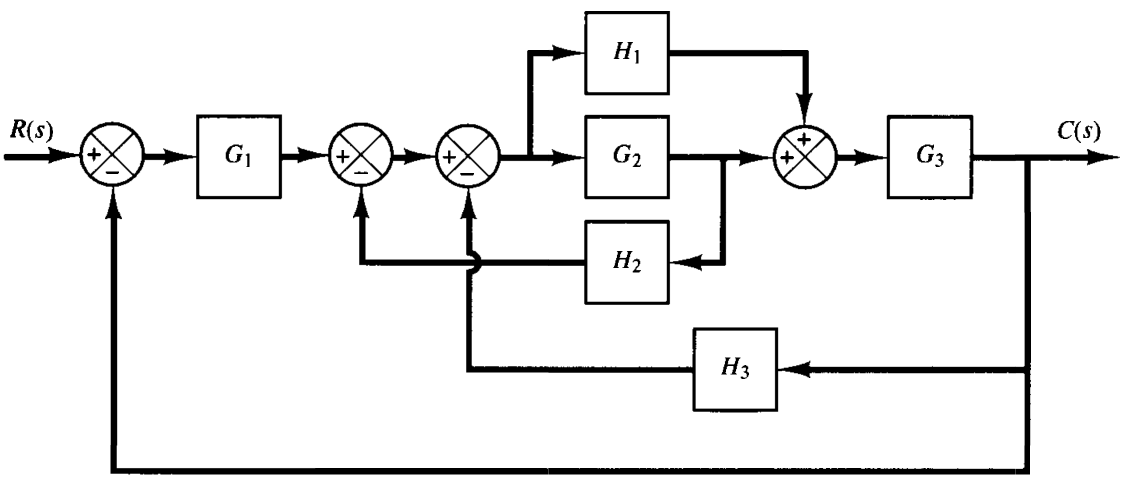

Figure 3–52 Block diagram of a system.

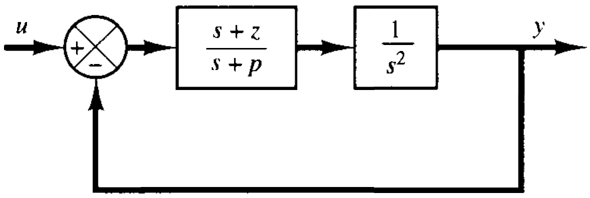

Figure 3–53 Control system.

B-3-6. Consider the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function of the system.

B-3-7. Obtain the transfer function $X_o(s)/X_i(s)$ of each of the three mechanical systems shown in Figure 3–54. In the diagrams, x_i denotes the input displacement and x_o denotes the output displacement. (Each displacement is measured from its equilibrium position.)

B-3-8. Obtain mathematical models of the mechanical systems shown in Figures 3–55(a) and (b).

B-3-9. Obtain a state-space representation of the mechanical system shown in Figure 3–56, where u_1 and u_2 are the inputs and y_1 and y_2 are the outputs.

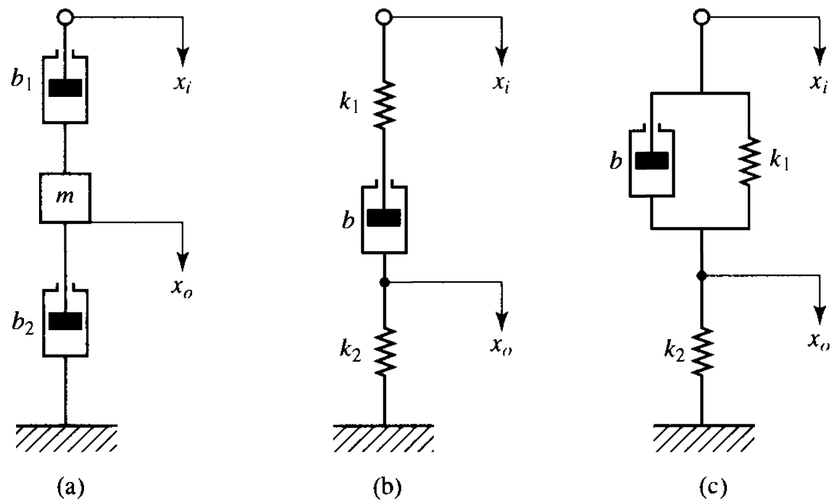

Figure 3–54 Mechanical systems.

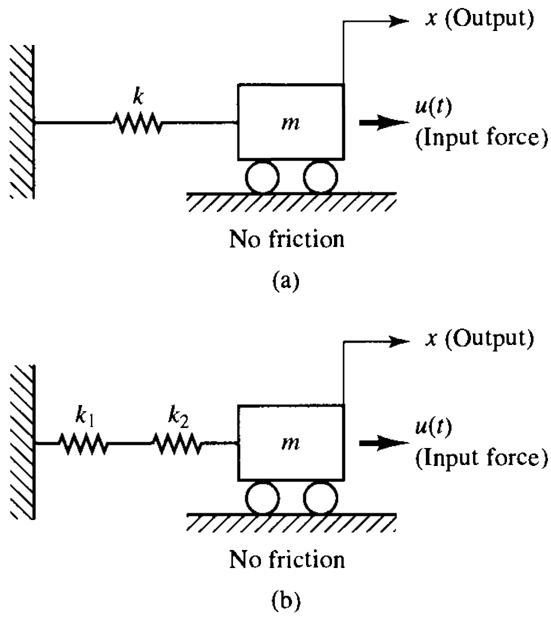

Figure 3–55 Mechanical systems.

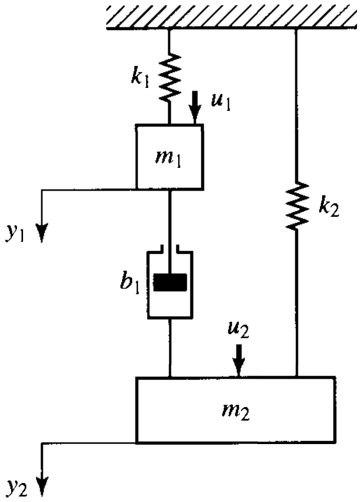

Figure 3–56
Mechanical system.

B-3-10. Consider the spring-loaded pendulum system shown in Figure 3–57. Assume that the spring force acting on the pendulum is zero when the pendulum is vertical, or $\theta = 0$. Assume also that the friction involved is negligible and the angle of oscillation θ is small. Obtain a mathematical model of the system.

B-3-11. Referring to Example 3–4, consider the inverted pendulum system shown in Figure 3–58. Assume that the mass of the inverted pendulum is m and is evenly distributed along the length of the rod. (The center of gravity of the pendulum is located at the center of the rod.) Assuming that θ is small, derive mathematical models for the system in the forms of differential equations, transfer functions, and state-space equations.

B-3-12. Derive the transfer function of the electrical system shown in Figure 3–59. Draw a schematic diagram of an analogous mechanical system.

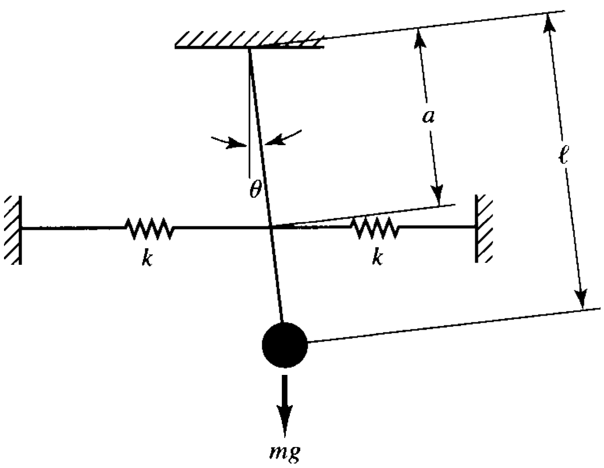

Figure 3–57 Spring-loaded pendulum system.

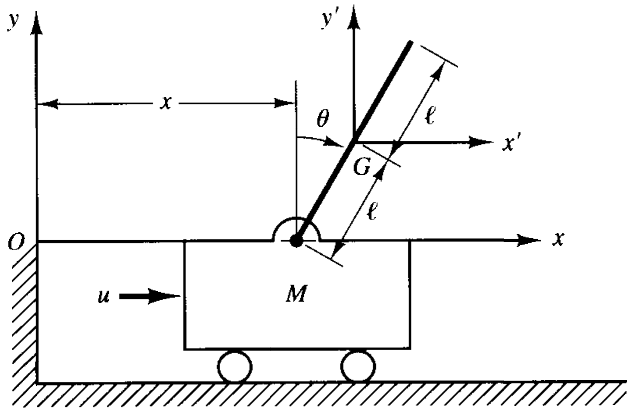

Figure 3–58 Inverted pendulum system.

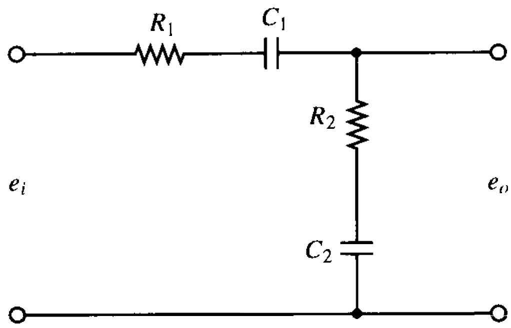

Figure 3–59 Electrical system.

B-3-13. Consider the liquid-level system shown in Figure 3–60. Assuming that $\bar{H} = 3$ m, $\bar{Q} = 0.02$ m³/sec, and the cross-sectional area of the tank is equal to 5 m², obtain the time constant of the system at the operating point $(\bar{H}, \bar{Q})$. Assume that the flow through the valve is turbulent.

B-3-14. Consider the conical water tank system shown in Figure 3–61. The flow through the valve is turbulent and is related to the head H by

$$Q = 0.005\sqrt{H}$$

where Q is the flow rate measured in m³/sec and H is in meters.

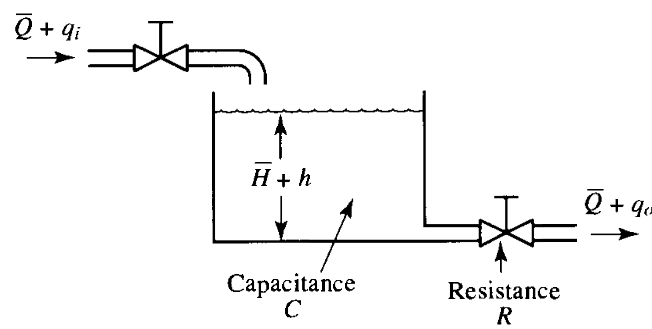

Figure 3–60 Liquid-level system.

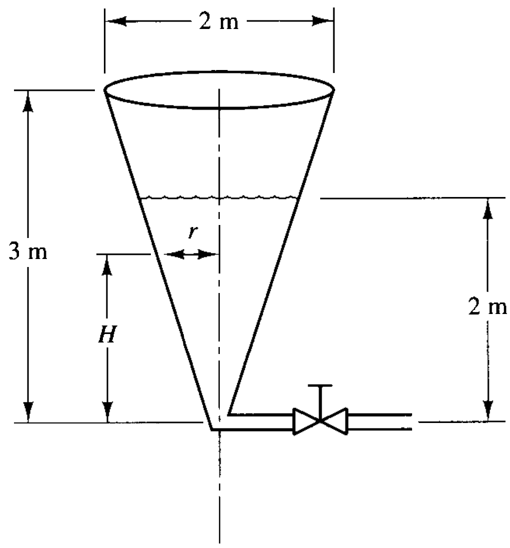

Figure 3–61 Conical water tank system.

Suppose that the head is 2 m at $t = 0$. What will be the head at $t = 60$ sec?

B-3-15. Consider the liquid-level system shown in Figure 3–62. At steady state the inflow rate is $\bar{Q}$ and the outflow rate is also $\bar{Q}$. Assume that at $t = 0$ the inflow rate is changed from $\bar{Q}$ to $\bar{Q} + q_i$, where q_i is a small quantity. The disturbance input is q_d, which is also a small quantity. Draw a block diagram of the system and simplify it to obtain $H_2(s)$ as a function of $Q_i(s)$ and $Q_d(s)$, where $H_2(s) = \mathscr{L}[h_2(t)]$, $Q_i(s) = \mathscr{L}[q_i(t)]$, and $Q_d(s) = \mathscr{L}[q_d(t)]$. The capacitances of tanks 1 and 2 are C_1 and C_2, respectively.

B-3-16. A thermocouple has a time constant of 2 sec. A thermal well has a time constant of 30 sec. When the thermocouple is inserted into the well, this temperature-measuring device can be considered a two-capacitance system.

Determine the time constants of the combined thermocouple–thermal well system. Assume that the weight of the thermocouple is 8 g and the weight of the thermal well is 40 g. Assume also that the specific heats of the thermocouple and thermal well are the same.

B-3-17. Suppose that the flow rate Q and head H in a liquid-level system are related by

$$Q = 0.002\sqrt{H}$$

Obtain a linearized mathematical model relating the flow rate and head near the steady-state operating point $(\bar{H}, \bar{Q})$, where $\bar{H} = 2.25$ m and $\bar{Q} = 0.003$ m³/sec.

B-3-18. Find a linearized equation for

$$y = 0.2x^3$$

about a point $\bar{x} = 2$.

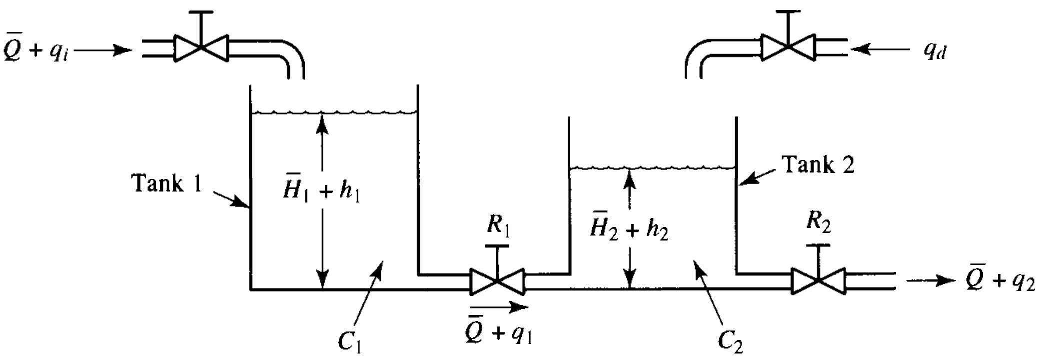

Figure 3–62 Liquid-level system.

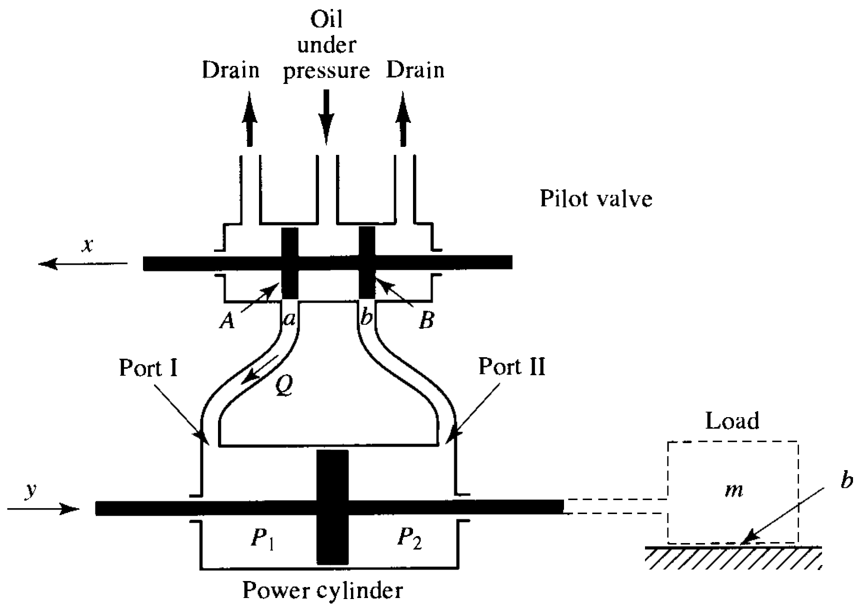

Figure 3–63 Schematic diagram of a hydraulic servomotor.

B-3-19. Linearize the nonlinear equation

$$z = x^2 + 4xy + 6y^2$$

in the region defined by $8 \le x \le 10, 2 \le y \le 4$.

B-3-20. Consider the hydraulic servomotor shown in Figure 3–63. Derive the transfer function $Y(s)/X(s)$. Assume that the inertia force due to the mass of power piston and shaft is negligible compared with the inertia force due to the load mass m and viscous friction force $b\dot{y}$.