What Do We Have for Today?

- Continue investigating performance specifications.
- Time domain exercises for second-order systems.
- Characteristic equation.
- Dominant poles and design issues.

Time Domain Exercises

- Consider

$$G(s) = \frac{1}{s^2 + 0.6s + 1}$$

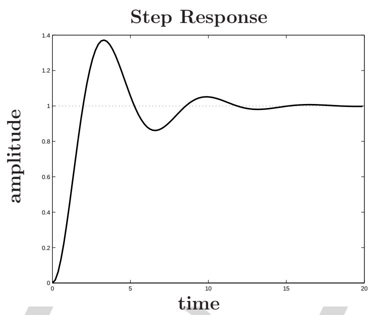
- Q : What is ω_n, ζ, ω_d and σ? How do these parameters affect the response?
- Octave and roots of a polynomial.
  Example. Determine the roots of $s^2 + 0.6s + 1$.

```
>> roots([1 0.6 1])
```


Time Domain Exercises

- Vary the damping factor ζ. Step response of $G(s)$.

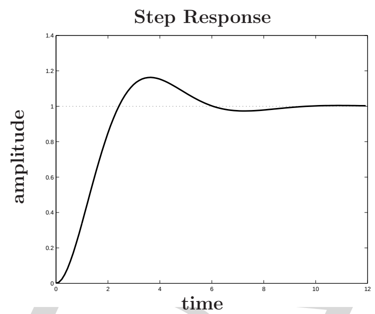
```
>> roots([1 0.6 1])

ans =

  -0.3000 + 0.9539i
  -0.3000 - 0.9539i

>> step(1, [1 0.6 1])
```



Time Domain Exercises

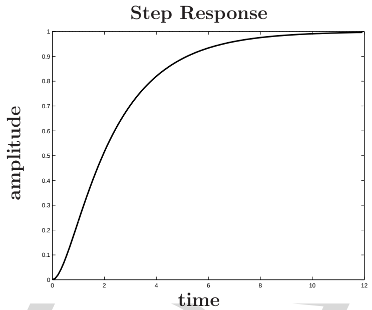
- Increase the damping.

```
>> roots([1 1 1])

ans =

  -0.5000 + 0.8660i
  -0.5000 - 0.8660i

>> step(1, [1 1 1])
```



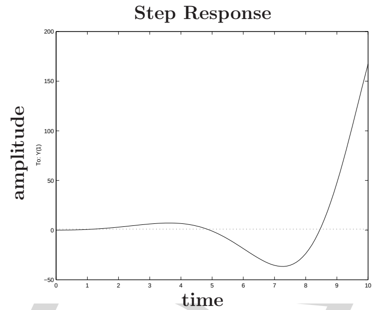
- **Overdamped.**

```
>> roots([1 2.5 1])

ans =

   -2.0000
   -0.5000

>> step(1, [1 2.5 1])
```



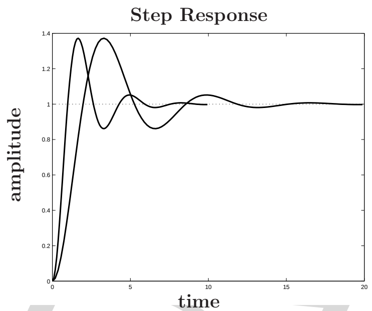
- **Negatively damped.**

```
>> roots([1 -1 1])

ans =

   0.5000 + 0.8660i
   0.5000 - 0.8660i

>> step(1, [1 -1 1], 0:0.1:10)
```



- **Increasing the natural frequency ω_n.**

```
>> wn = 1; z = 0.3;
>> step(wn^2, [1 2*z*wn wn^2])
>> hold
Current plot held
>> wn = 2;
>> step(wn^2, [1 2*z*wn wn^2])
```

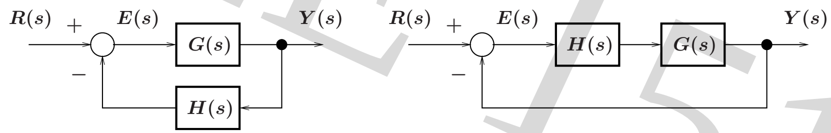


Characteristic Equation

- **Error equation.**

$$E(s) = \frac{1}{1 + H(s)G(s)} R(s)$$

- **We have the same error equation for these two systems.**



- **Error equation and sensitivity function : $E = SR$.**

- Poles of the transfer function from $R(s)$ to $E(s)$.
  $\Rightarrow$ roots or zeros of $1 + H(s)G(s)$.

- Characteristic equation : $1 + H(s)G(s) = 0$.

- Example.

$$G(s) = \frac{1}{s^2 + 0.6s + 1}, \qquad H(s) = 1$$

- Error equation and characteristic equation.

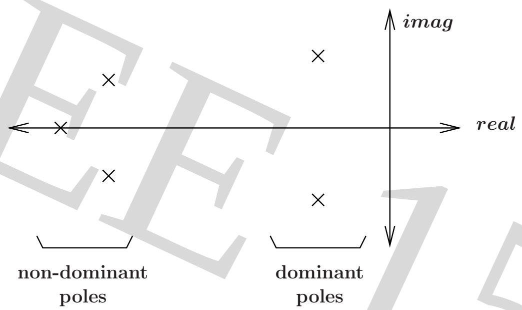
$$\frac{E(s)}{R(s)} = \frac{s^2 + 0.6s + 1}{s^2 + 0.6s + 2}$$

$$1 + H(s)G(s) = 0 \quad \Rightarrow \quad s^2 + 0.6s + 2 = 0$$

roots without feedback : $s = -0.3 \pm j0.9539$
roots with feedback : $s = -0.3 \pm j1.3820$

- Therefore, feedback
  - modifies system behavior.
  - moves the location of the poles.

- Location of poles on the s-plane.



- dominant poles : decay slowly.
- non-dominant poles : decay quickly.

- In some cases, we need to worry only about dominant poles and ignore non-dominant poles.

- Example. Compare the response of

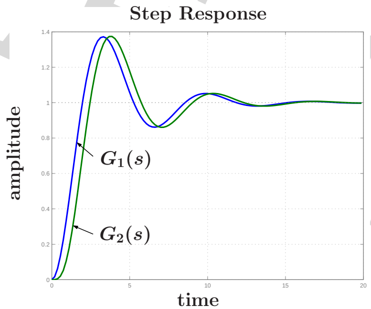
$$G_1(s) = \frac{1}{s^2 + 0.6s + 1}$$

and

$$G_2(s) = G_1(s)\frac{9}{s^2 + 4.2s + 9}$$

- Step responses.

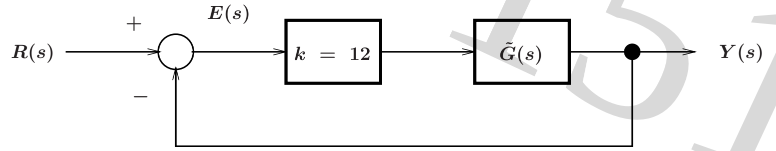
```
>> n1 = 1; d1 = [1 0.6 1]; step(n1, d1)
>> [n2, d2] = series(n1, d1, 9, [1 4.2 9]);
>> step(n2, d2)
```



- What happens if you ignore 'insignificant poles?'

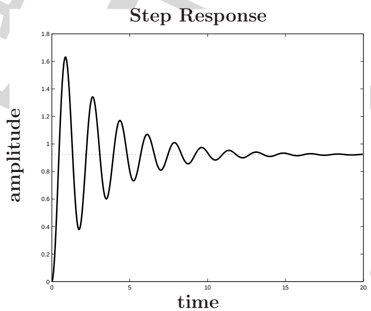
  Suppose the actual plant model is $G(s) = G_2(s)$. However, in designing the control, non-dominant poles where ignored, i.e., we use $\tilde{G}(s) = G_1(s)$.

- Design the feedback system.



- Step response of the paper design with the plant $\tilde{G}(s)$.

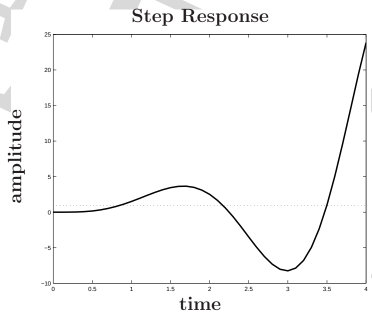
```
>> [nc1, dc1] = cloop(12*n1, d1, -1);
>> step(nc1, dc1)
```



$\Rightarrow$ We have a stable system.

- Check using the actual plant model, $G(s)$.

```
>> [nc2, dc2] = cloop(12*n2, d2, -1);
>> step(nc2, dc2, [0:0.1:4])
```



$\Rightarrow$ The system is unstable.

Dominant Poles

- The step responses of low-order systems and high-order systems may be similar.

- Good thing. This can simplify the determination of the step response of a high-order system.

  Inspect the T_r, T_d, natural frequency, etc.

  There is 'little' difference between the step responses of the second-order system $G_1(s)$ and the fourth-order system $G_2(s)$.

Dominant Poles

- Not so good thing. This can lead to incorrectly identifying the system order, and the system model.

  Inspect the T_r, T_d, natural frequency, etc.

  In designing a control system for $G_2(s)$, one may incorrectly model the system as $G_1(s)$, and thus lead to a poorly designed controller.

- An accurate model is important in control systems design.

Summary

- Varying ζ and ω_n varies the response of a second-order system.

- Characteristic equation, and how it describes a system.

- Dominant poles. Making the right choice.