# Today's EEE 151 Lecture

- Comparison of block diagrams and SFGs.
- SFG transformations.
- Mason gain rule.
- Summary.






# Signal Flow Graphs

- Input-output relationship depicted by line segments between input and output nodes.


- Signal flow is unidirectional from input to output.
- Simpler representation of a system compared to block diagrams.






# Signal Flow Graphs

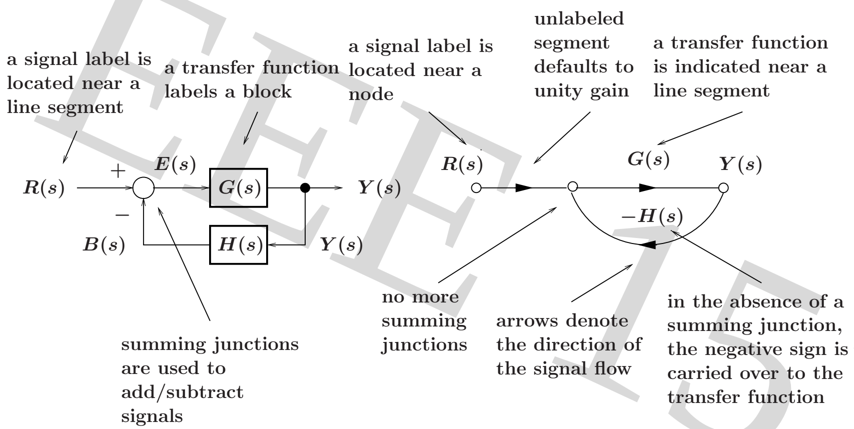
- Complex systems are represented by multiple and interconnected graphs.
- Complex SFGs may be reduced by SFG transformations.
- Mason gain rule can also be applied to simplify SFGs.





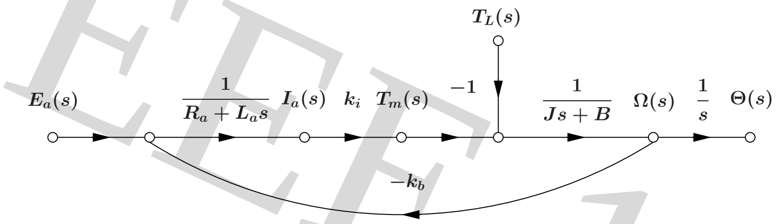

# Signal Flow Graphs

- Comparison.



## Signal Flow Graphs

- Example. SFG of the DC servomotor system.




where

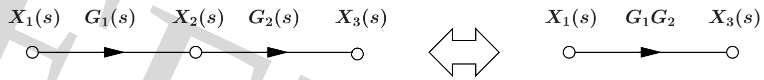
$$\Omega(s) = \mathscr{L}[\dot{\theta}(t)]$$
$$\Theta(s) = \mathscr{L}[\theta(t)]$$



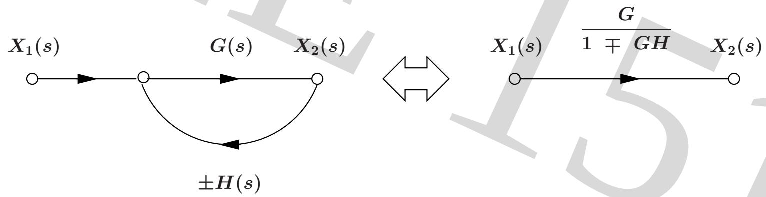



## SFG Transformations

- Series connected blocks




- Feedback loop









## Mason Gain Rule

- Mason gain rule

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

where

$y_{in}$ = input node variable
$y_{out}$ = output node variable
$M$ = gain between $y_{in}$ and $y_{out}$
$N$ = number of forward paths
$M_k$ = gain of $k$-th forward path
$\Delta$ = determinant of the whole SFG
$\Delta_k$ = $\Delta$ for the SFG which is non-touching with the $k$-th forward path.






## Mason Gain Rule

- Computation of the determinant $\Delta$.

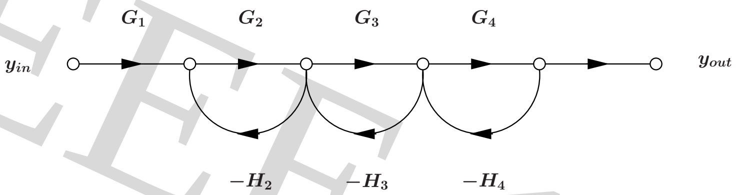
$$\Delta = 1 - \sum_m P_m^1 + \sum_m P_m^2 - \sum_m P_m^3 + \ldots$$

where $P_m^r$ is the gain product of the $m$-th combination of $r$ non-touching loops.

- Two parts of the SFG are non-touching if they do not share a common node.

## Mason Gain Rule

- Example. SFG reduction using Mason gain rule.

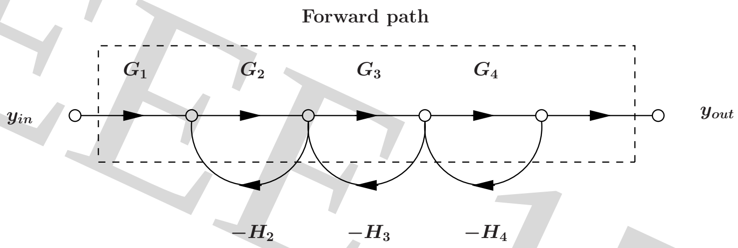



- The SFG has
  - one forward path.
  - three individual loops.
  - one pair of non-touching loops.




## Mason Gain Rule

- Forward path.

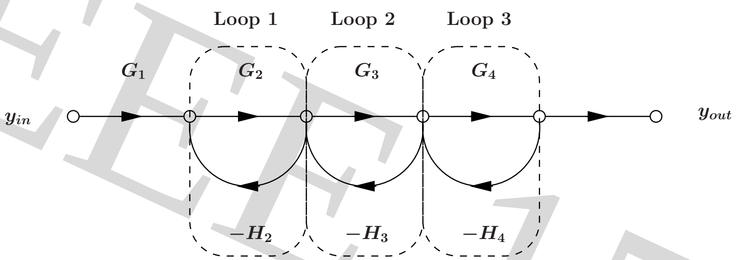



$$M_1 = G_1G_2G_3G_4$$
$$\Delta_1 = 1$$




## Mason Gain Rule

- Individual loops.

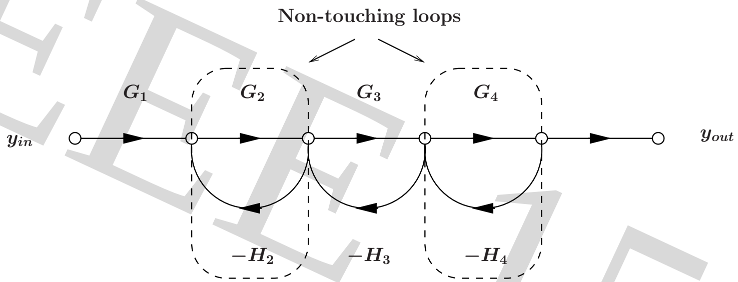



$$\text{Loop 1}: P_1^1 = -G_2H_2$$
$$\text{Loop 2}: P_2^1 = -G_3H_3$$
$$\text{Loop 3}: P_3^1 = -G_4H_4$$




## Mason Gain Rule

- Pairs of non-touching loops.




$$P_1^2 = (-G_2H_2)(-G_4H_4)$$
$$= G_2H_2G_4H_4$$

- **Determinant of the SFG.**

$$\Delta = 1 - (P_1^1 + P_2^1 + P_3^1) + P_1^2$$

- **Transfer function**

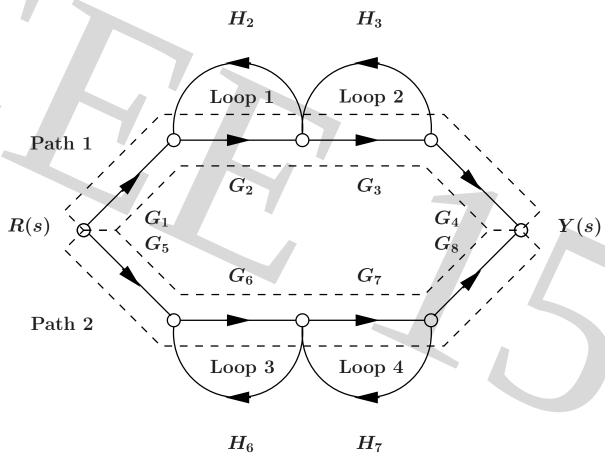
$$\frac{y_{out}}{y_{in}} = \frac{M_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{1 + G_2H_2 + G_3H_3 + G_4H_4 + G_2H_2G_4H_4}$$







- **Example. Interacting system.**








- **Two forward paths from input to output.**

$$M_1 = G_1G_2G_3G_4$$
$$M_2 = G_5G_6G_7G_8$$

- **Individual loops.**

$$P_1^1 = G_2H_2, \quad P_2^1 = G_3H_3,$$
$$P_3^1 = G_6H_6, \quad P_4^1 = G_7H_7$$





- **Pairs of non-touching loops.**
  **For conciseness, let**

$$\text{Loop 1}: L_1 = G_2H_2, \quad \text{Loop 2}: L_2 = G_3H_3,$$
$$\text{Loop 3}: L_3 = G_6H_6, \quad \text{Loop 4}: L_4 = G_7H_7$$

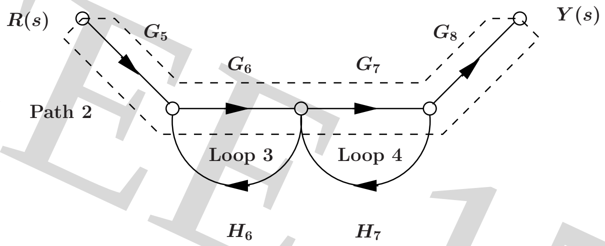
**Then,**

$$P_1^2 = L_1L_3, \quad P_2^2 = L_1L_4,$$
$$P_3^2 = L_2L_3, \quad P_4^2 = L_2L_4$$

- **Determinant of the SFG.**

$$\Delta = 1 - (P_1^1 + P_2^1 + P_3^1 + P_4^1) + (P_1^2 + P_2^2 + P_3^2 + P_4^2)$$

- With the upper forward path removed,



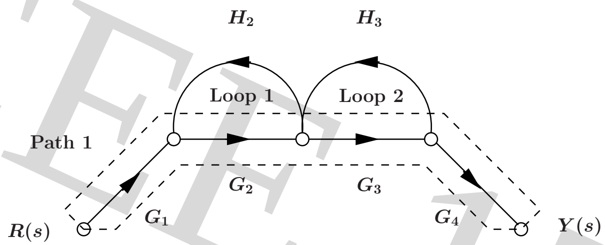

only Loop 3 and Loop 4 remain. Thus,

$$\Delta_1 = 1 - (L_3 + L_4)$$






- With the lower forward path removed,




only Loop 1 and Loop 2 remain. Thus,

$$\Delta_2 = 1 - (L_1 + L_2)$$






- Transfer function

$$\frac{Y(s)}{R(s)} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta} = \frac{\left\{ \begin{array}{l} G_1G_2G_3G_4(1 - L_3 - L_4) + \\ G_5G_6G_7G_8(1 - L_1 - L_2) \end{array} \right\}}{\left\{ \begin{array}{l} 1 - L_1 - L_2 - L_3 - L_4 + \\ L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4 \end{array} \right\}}$$

- Note : Mason gain rule requires some care.




## Summary

- What are SFGs?
  How are they different from block diagrams?
- Mason gain rule; an important reduction technique.
- How do you apply Mason gain rule?