Upcoming Lecture Features

- Main things we will be looking at.
  - block diagrams
  - signal flow graphs
  - Mason gain rule
  - general control system

- For today's lecture, we have
  - block diagrams
  - interpreting a transfer function

What is a Transfer Function?

- Dynamic systems are represented by differential equations.

- Laplace transformation simplifies solutions to differential equations.

- In control systems, variables are controlled to achieve a specific or desired behavior.

What is a Transfer Function?

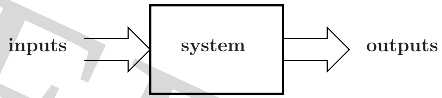
- Relationship between controlling variable(s) and controlled variable(s) are needed.

- Relationship usually represented by a transfer function.

- Controlling variable - input
  Controlled variable - state, output
  Transfer function - mapping of input to output of a dynamic system.

Block Diagrams

- Input-output relationship usually depicted by blocks.



- Signal flow is unidirectional from input to output.

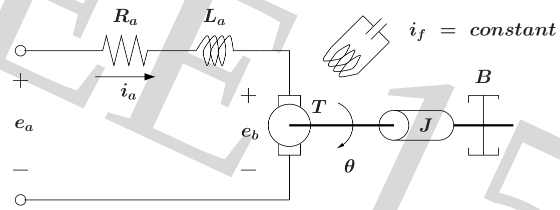
- Simple systems can be represented by a single block.

- Complex systems are represented by multiple and interconnected blocks.

  Room heating example.
  Hard to model the system as one complex system.
  Break it down into simpler subsystems.

- Block diagrams may be reduced by block diagram transformations.
  - series connected
  - single feedback
  - parallel connected

- Example. DC servomotor.
  - small rotor inertia
  - high torque-to-inertia ratio



- Focus on armature-current control.

- Electrical equations.

$$e_a = R_a i_a + L_a \dot{i}_a + e_b \quad \text{(KVL)}$$
$$e_b = k_b \dot{\theta} \quad \text{(back-EMF)}$$

- Mechanical equations.

$$T = k_i i_a \quad \text{(motor torque)}$$
$$T = J\ddot{\theta} + B\dot{\theta} \quad \text{(sum torques)}$$

- Variables.
  input (e_a) $\Rightarrow$ state ($\dot{\theta}$ and i_a) $\Rightarrow$ output (θ)

- From KVL equation,

$$e_a = R_a i_a + L_a \dot{i}_a + e_b$$
$$\Rightarrow R_a i_a + L_a \dot{i}_a = e_a - e_b$$
$$\mathscr{L} \Rightarrow R_a I_a + L_a s I_a = E_a - E_b \quad \text{(ignore I.C.)}$$
$$\Rightarrow I_a = \frac{1}{R_a + sL_a}(E_a - E_b)$$

- From back-EMF equation,

$$e_b = k_b \dot{\theta}$$
$$\mathscr{L} \Rightarrow E_b = k_b \Omega$$

Block Diagrams

- From motor torque equation,

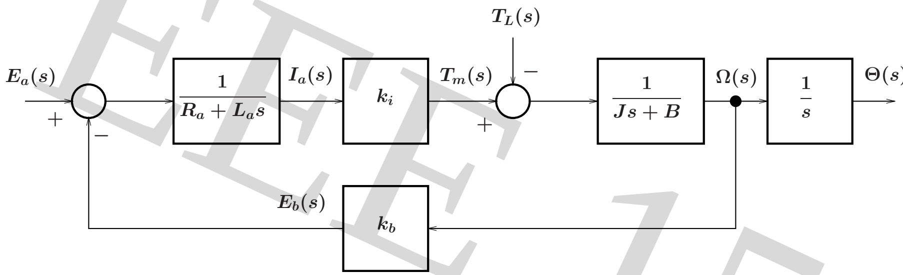
$$\begin{aligned} T_m &= k_i i_a \\ \mathscr{L} \Rightarrow T_m &= k_i I_a \end{aligned}$$

- From summation of torques equation,

$$\begin{aligned} T_m &= J\ddot{\theta} + B\dot{\theta} + T_L \\ \Rightarrow T_m - T_L &= J\ddot{\theta} + B\dot{\theta} = J\dot{\omega} + B\omega \\ \mathscr{L} \Rightarrow T_m - T_L &= Js\Omega + B\Omega \\ \Rightarrow \Omega &= \frac{1}{sJ + B}(T_m - T_L) \end{aligned}$$

Block Diagrams

- Block diagram of a DC servomotor system

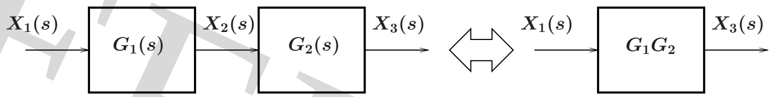


where

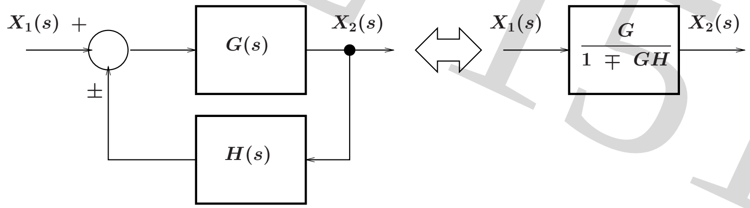
$$\begin{aligned} \Omega(s) &= \mathscr{L}[\dot{\theta(t)}] \\ \Theta(s) &= \mathscr{L}[\theta(t)] \end{aligned}$$

Block Diagram Transformations

- Series connected blocks

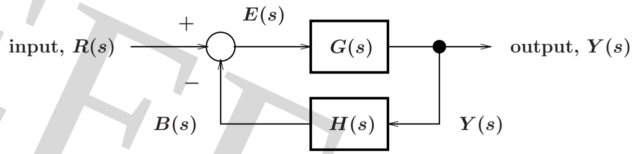


- Feedback loop



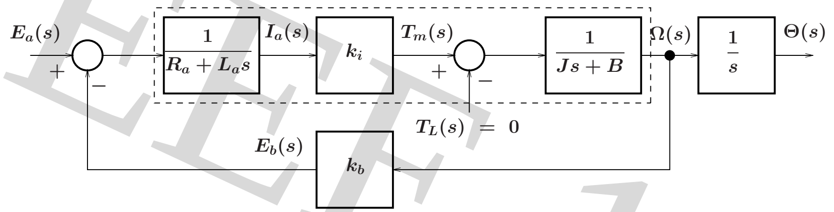
Block Diagram Transformations

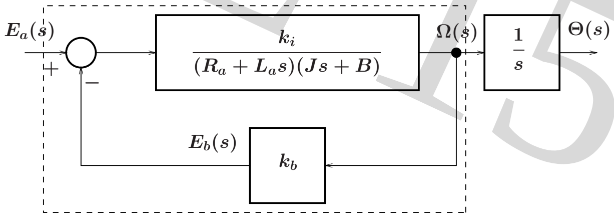
- Closed-loop transfer function



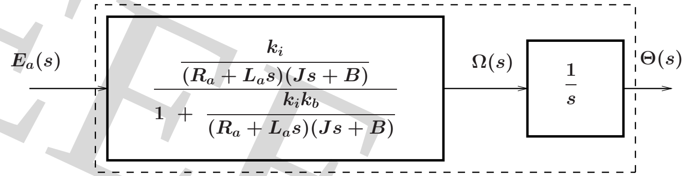
$$\begin{aligned} \text{error equation :}\quad E(s) &= R(s) - B(s) \\ \Rightarrow E(s) &= R(s) - H(s)Y(s) \\ \text{output equation :}\quad Y(s) &= G(s)E(s) \\ \Rightarrow Y(s) &= G(s)[R(s) - H(s)Y(s)] \\ \text{solving for } Y(s) :\quad Y(s)[1 + G(s)H(s)] &= G(s)R(s) \\ \Rightarrow \frac{Y(s)}{R(s)} &= \frac{G(s)}{1 + G(s)H(s)} \end{aligned}$$

- Example. DC servomotor with no load (i.e., $T_L = 0$).

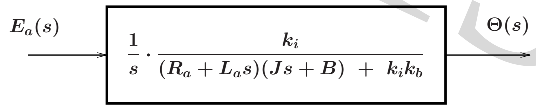




- Further reduction ...



- Transfer function of a DC servomotor



Summary

- There is more than one way to represent a system.

  The choice depends on what you want to do.

- Transfer function representation.

  One input, one output representation. Very useful in control system feedback design.

- Block diagrams are also used to represent systems.

  You usually come up with block diagrams first, then simplify to a transfer function.