Today's EEE 151 Lecture

- Mathematical modeling of dynamic systems.
- What is a model? What is a dynamic system?
- Systems we will be looking at.
  - electrical systems
  - mechanical systems (translational and rotational)
  - electromechanical systems
  - thermal systems and liquid-level systems

What is a Model?

- Dictionary definition. Model.
  - n. A small object, usually built to scale, that represents in detail another, often larger object.
  - n. A preliminary work or construction that serves as a plan from which a final product is to be made.
  - n. A schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics.
  - n. A style or design of an item.
  - n. One serving as an example to be imitated or compared.

What is a Model?

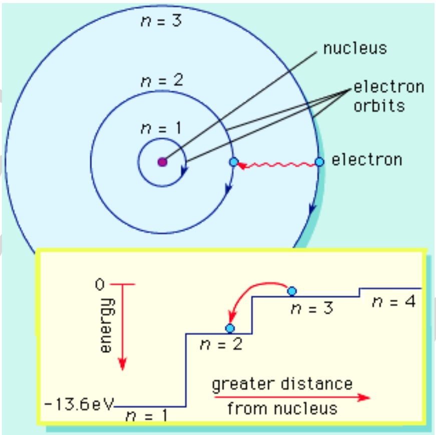
- Other definitions of a model.
  - n. A person employed to display merchandise, such as clothing or cosmetics.
  - n. Zoology. An animal whose appearance is copied by a mimic.
- Modeling (computer terms).
  Generally, the process of representing a real-world object or phenomenon as a set of mathematical equations. More specifically, the term is often used to describe the process of representing 3-D objects in a computer.

What is a Model?

- Verbal model.

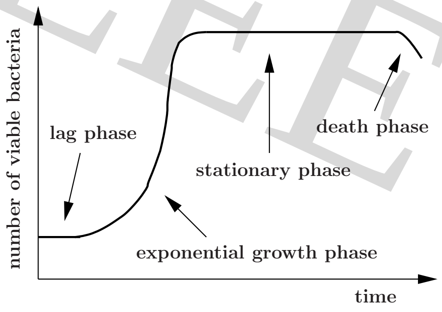
  Bohr atomic model.

  Bohr proposed that electrons are restricted to certain fixed (quantized) orbits. An electron can jump, suddenly, between these orbits by absorbing or emitting a photon of the appropriate precise wavelength.



- Graphical model.

  Bacteria growth model.



The growth curve shown is associated with simplistic conditions known as a batch culture.

Single bacterial culture introduced into and growing in a fixed volume with a fixed (limited) amount of nutrient.

- Mathematical model.

  Radioactive decay.
  In 1903, Rutherford and Soddy proposed the law of radioactive change which states that rate of decay of radioactive matter was proportional to the amount present. Mathematically, we say

$$\text{rate of change of } [A] = -k \cdot [A]$$

where $[A]$ and k are the concentration and the rate of decay of substance A, respectively. Thus,

$$[A](t) = [A](0)b^t \qquad 0 < b < 1$$

- We will be using mathematical models in EEE 151.

  We have tools (differential equations and Laplace transforms) to deal with mathematical equations.

- May sometimes start from a verbal model or a graphical model and extract an approximate mathematical model.

- Other control techniques exist for directly dealing with verbal and graphical models.

Dynamic Systems

- Systems where system variables (state variables) change with respect to time.

- Dynamic systems are usually modeled with differential equations (or possibly, difference equations).

  Example. Robotic manipulator model.

$$D(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau$$

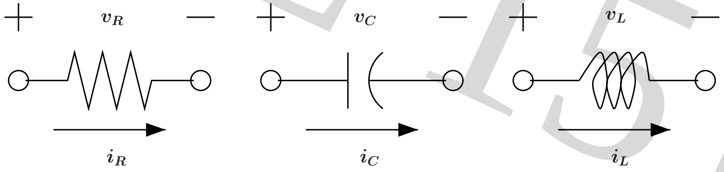
- We need to able to handle differential equations.

Electrical Systems

- Variables : voltage v and current i.

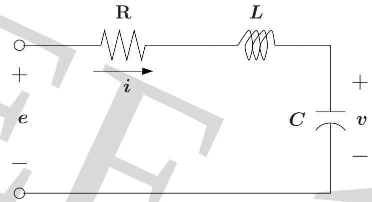
- Resistance, capacitance and inductance.

$$v_R = i_R R, \quad i_C = C\frac{dv_C}{dt}, \quad v_L = L\frac{di_L}{dt}$$



Electrical Systems

- Example. RLC circuit



Two first-order differential equations

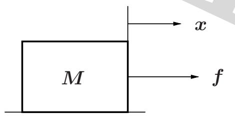
$$i = C\frac{dv}{dt} \Rightarrow i = C\dot{v}$$

$$e = iR + L\frac{di}{dt} + v \Rightarrow e = iR + L\dot{i} + v$$

Mechanical Systems

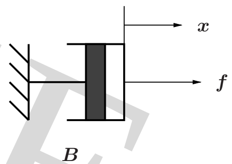
- Translational variables :
  - position, x
  - velocity, $\dot{x} = \frac{dx}{dt}$
  - force, f

- Mass : $f = M\ddot{x}$

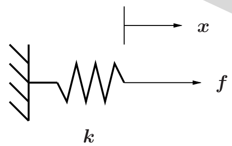


Mechanical Systems

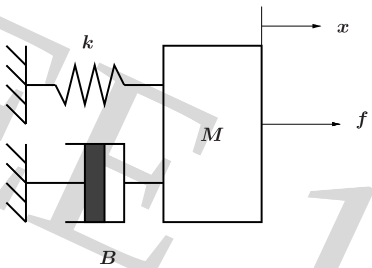
- Friction (viscuous): $f = B\dot{x}$



- Spring (linear): $f = kx$



- Example. Mass, spring and damper system



Summing forces along the horizontal

$$f = M\ddot{x} + kx + B\dot{x}$$

- Rotational

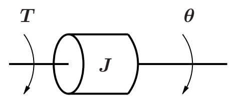
  Variables :

  – angle, θ

  – angular velocity, $\dot{\theta}$

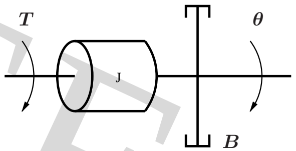
  – torque, T

- Inertia (rotational) : $T = J\ddot{\theta}$

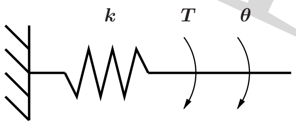


for a circular disk rotating about a geometric axis $J = \frac{1}{2}Mr^2$

- Friction (rotational): $T = B\dot{\theta}$

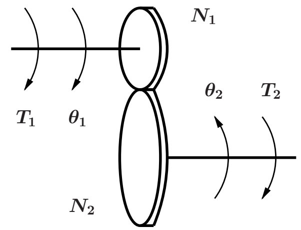


- Spring (torsional): $T = k\theta$



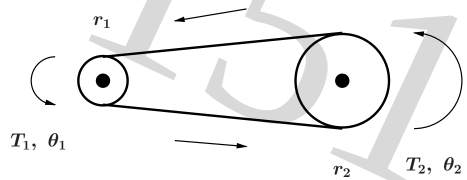
- Gears.

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\dot{\theta}_2}{\dot{\theta}_1}$$

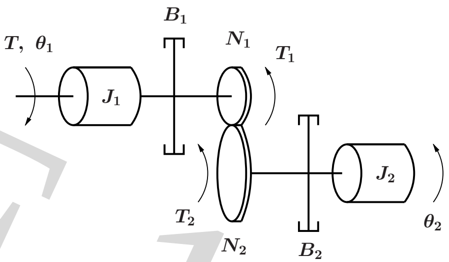


- Belt-driven gears.

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{r_1}{r_2}$$



- Example. Reflected load



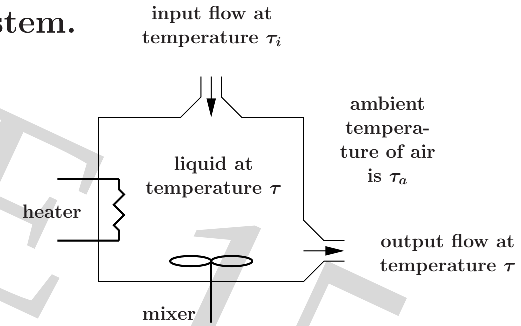
T : applied externally
T_1 : applied by gear 2
T_2 : applied to shaft 2

$$T_2 = J_2\ddot{\theta}_2 \ + \ B_2\dot{\theta}_2$$
$$T = J_1\ddot{\theta}_1 \ + \ B_1\dot{\theta}_1 \ + \ T_1$$

$$\Rightarrow T_1 \ = \ \frac{N_1}{N_2}T_2 \ = \ \left(\frac{N_1}{N_2}\right)^2 J_2\ddot{\theta}_1 \ + \ \left(\frac{N_1}{N_2}\right)^2 B_2\dot{\theta}_1$$

- Another modeling example. Temperature control system.

  Liquid is flowing out at a certain rate while being replaced by liquid with temperature τ_i.



- The liquid is heated by an electric heater and agitated by a mixer such that the liquid temperature is uniform inside the tank.

- Define

  $q_e(t) \triangleq$ heat increase supplied by the heater.
  $q_i(t) \triangleq$ heat increase from entering liquid.
  $q_l(t) \triangleq$ heat absorbed by the liquid.
  $q_o(t) \triangleq$ heat decrease from exiting liquid.
  $q_s(t) \triangleq$ heat loss through tank surface.

- Conservation of energy.

$$q_i(t) \ + \ q_e(t) \ = \ q_l(t) \ + \ q_o(t) \ + \ q_s(t)$$

- The heat absorbed by the liquid is related to the thermal capacity C and the temperature change by

$$q_l(t) \ = \ C\frac{d\tau(t)}{dt}$$

- Let $v(t)$ be the liquid flow rate in (and out) of the tank. If H is the specific heat of the liquid, then

$$q_i(t) \ = \ v(t)H\tau_i(t) \text{ and } q_o(t) \ = \ v(t)H\tau(t)$$

Thermal Systems and Liquid-level Systems

- The heat loss through the tank surface in terms of the thermal resistance R of the tank surface is

$$q_s(t) = \frac{\tau(t) - \tau_a(t)}{R}$$

- Combining the above equations, we get the differential equation model of the system.

$$q_e(t) + v(t)H\tau_i(t) = C\frac{d\tau(t)}{dt} + v(t)H\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

Thermal Systems and Liquid-level Systems

- Assuming that the flow rate $v(t)$ is at a constant value V, we get a time-invariant first-order ODE.

$$q_e(t) + VH\tau_i(t) = C\frac{d\tau(t)}{dt} + VH\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

- In view of a control system,
  - $q_e(t)$ is the input,
  - $\tau_i(t)$ and $\tau_a(t)$ are disturbances, and
  - $\tau(t)$ as the output.

Why Use a Model?

- A mathematical model can be
  - used to simulate hypothetical situations,
  - subjected to states that would be dangerous in reality, and
  - used as basis for synthesizing controllers.

- Note. Real processes are complex, and getting an exact model is usually impossible.

  However, feedback is lenient (to some degree) and controllers can be designed using simple models.

  The model must capture the essential features of the plant.

Why Use a Model?

- Some definitions.
  - nominal model.
    An approximate description of the plant used for control system design.
  - calibration model.
    A comprehensive description of the plant. It can include behaviors not used for control design but may impact performance.
  - model error.
    Difference between the nominal model and the calibration model. This error may be unknown but bounds may be available.

Why Use a Model?

- The nominal model may be a simplified version of the calibration model.
- The nominal model is used in doing the control.

  The value of the plant input is directly based on this model.
- The calibration model is used to verify if the control works.

  This model may be used in numerical simulations to see if the controller performance is satisfactory.

- Example. Robotic manipulator model.

$$D(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau$$

  - $D(q)\ddot{q}$ inertia term.
  - $C(q,\dot{q})$ coriolis and centrifugal terms.
  - $G(q)$ gravity term.
- For control purposes, $C(q,\dot{q})$ term may be neglected.

  In some instances, only the gravity term is used in generating the control.
- When doing simulations, the complete model is used.

Summary

- What is a dynamic system?

  What are models? What is a mathematical model?
- Familiarize with
  - electrical systems
  - mechanical systems (translational and rotational)
- Review
  - electromechanical systems
  - thermal systems and liquid-level systems