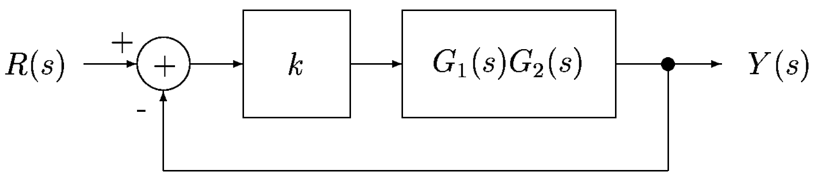
EEE 101 AY2001-2002 : Second Exam Sample Problems

1. Given

$$G_1(s) \;=\; \frac{A_1}{s \,+\, a}, \qquad G_2(s) \;=\; \frac{A_2}{s^2 \,+\, 2\zeta\omega_n s \,+\, w_n^2}, \qquad a,\ \zeta,\ \omega_n \;>\; 0$$

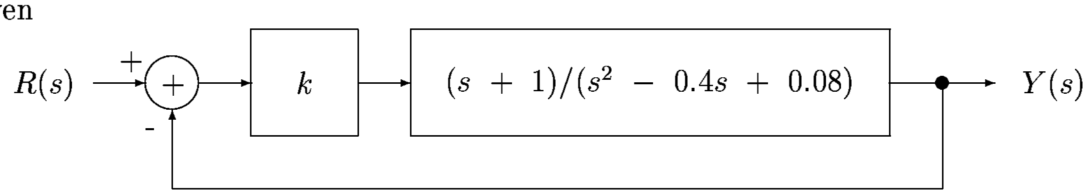
a. Determine A_1 such that the DC gain of the open-loop transfer function $G_1(s)$ is unity. Determine A_2 such that the DC gain of $G_2(s)$ is also unity.

b. Using A_1 and A_2 specified in (a), determine the characteristic equation for the following closed loop unity gain feedback system. Specify your answer in the form $a_0 s^n \,+\, a_1 s^{n-1} \,+\, \ldots \,+\, a_n \,=\, 0$.



c. Use RH to determine the range of $k \;>\; 0$ such that the system in (b) is stable.

2. Given



a. Sketch the root locus.

b. Determine the value of gain k at the breakaway point.

c. What are the poles of the above system at the gain k specified in (b)?

d. Determine the value of gain k that will make above system is marginally stable.

e. What are the poles of the above system at the gain k specified in (d).

3. For the second-order system $G_2(s)$ in item 1,

a. Show analytically that the graph of the poles of $G_2(s)$ on the s-plane for constant-ω_n is a semicircle of radius r. Specify r in terms of ω_n.

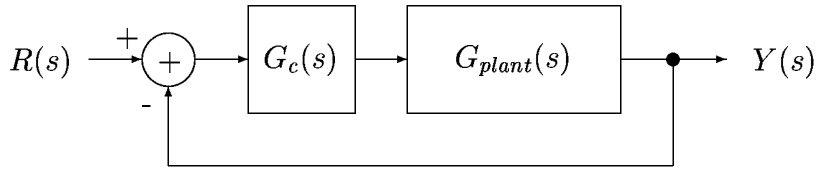
b. Show analytically that the graph of the poles of $G_2(s)$ for constant-ζ are two rays from the origin of the s-plane. Specify θ (the angle from the positive real axis to the ray in the second quadrant) in terms of ζ.

c. Show which region in the s-plane corresponds to $\omega_n \;>\; 0.1$ and $\zeta \;>\; 1/\sqrt{2}$.

4. For the first-order system $G_1(s)$ in item 1,

a. Derive an expression for the rise time T_r of $G_1(s)$ in terms of a. Your answer should contain exactly one logarithmic expression.
b. Derive an expression for the delay time T_d of $G_1(s)$ in terms of a. Your answer should contain exactly one logarithmic expression.

5. Welding control. Given the following block diagram for a welding control system



The controller transfer function $G_c(s)$ and the plant transfer function $G_{plant}(s)$ are

$$G_c(s) = k\frac{s + a}{s + 1}, \qquad G_{plant}(s) = \frac{1}{s(s + 2)(s + 3)}$$

where k and a are controller parameters.

a. Determine the characteristic equation for the welding control system in terms of the controller parameters k and a. Specify your answer in the form $a_0 s^n + a_1 s^{n-1} + \ldots + a_n = 0$.
b. Using RH, determine the ranges of k and a where the system is stable.