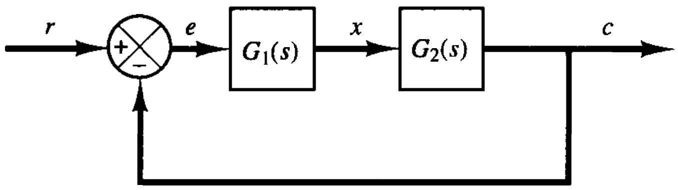

Figure 8–132
Control system.

Noting that there is another corner frequency at $\omega = 0.5$ rad/sec and the slope of the magnitude curve in the low-frequency region is -40 dB/decade, $G(j\omega)$ can be tentatively determined as follows:

$$G(j\omega) = \frac{K\left(\frac{j\omega}{0.5} + 1\right)}{(j\omega)^2\left[\left(\frac{j\omega}{2}\right)^2 + 0.1(j\omega) + 1\right]}$$

Since from Figure 8–131 we find $|G(j0.1)| = 40$ dB, the gain value K can be determined as unity. Also, the calculated phase curve, $\angle G(j\omega)$ versus ω, agrees with the given phase curve. Hence, the transfer function $G(s)$ can be determined as

$$G(s) = \frac{4(2s + 1)}{s^2(s^2 + 0.4s + 4)}$$

A-8-25. A closed-loop control system may include an unstable element within the loop. When the Nyquist stability criterion is to be applied to such a system, the frequency-response curves for the unstable element must be obtained.

How can we obtain experimentally the frequency-response curves for such an unstable element? Suggest a possible approach to the experimental determination of the frequency response of an unstable linear element.

Solution. One possible approach is to measure the frequency-response characteristics of the unstable element by using it as a part of a stable system.

Consider the system shown in Figure 8–132. Suppose that the element $G_1(s)$ is unstable. The complete system may be made stable by choosing a suitable linear element $G_2(s)$. We apply a sinusoidal signal at the input. At steady state, all signals in the loop will be sinusoidal. We measure the signals $e(t)$, the input to the unstable element, and $x(t)$, the output of the unstable element. By changing the frequency [and possibly the amplitude for the convenience of measuring $e(t)$ and $x(t)$] of the input sinusoid and repeating this process, it is possible to obtain the frequency response of the unstable linear element.

PROBLEMS

B-8-1. Consider the unity-feedback system with the open-loop transfer functions.

$$G(s) = \frac{10}{s + 1}$$

Obtain the steady-state output of the system when it is subjected to each of the following inputs:

(a) $r(t) = \sin(t + 30°)$
(b) $r(t) = 2\cos(2t - 45°)$
(c) $r(t) = \sin(t + 30°) - 2\cos(2t - 45°)$

B-8-2. Consider the system whose closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K(T_2 s + 1)}{T_1 s + 1}$$

Obtain the steady-state output of the system when it is subjected to the input $r(t) = R\sin\omega t$.

B-8-3. Sketch the Bode diagrams of the following three transfer functions:

(a) $G(s) = \dfrac{T_1 s + 1}{T_2 s + 1} \qquad (T_1 > T_2 > 0)$

(b) $G(s) = \dfrac{T_1 s - 1}{T_2 s + 1} \qquad (T_1 > T_2 > 0)$

(c) $G(s) = \dfrac{-T_1 s + 1}{T_2 s + 1} \qquad (T_1 > T_2 > 0)$

B-8-4. Plot the Bode diagram of

$$G(s) = \frac{10(s^2 + 0.4s + 1)}{s(s^2 + 0.8s + 9)}$$

B-8-5. Given

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

show that

$$|G(j\omega_n)| = \frac{1}{2\zeta}$$

B-8-6. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{s + 0.5}{s^3 + s^2 + 1}$$

This is a nonminimum-phase system. Two of the three open-loop poles are located in the right-half s plane as follows:

$$\text{Open-loop poles at } s = -1.4656$$
$$s = 0.2328 + j0.7926$$
$$s = 0.2328 - j0.7926$$

Plot the Bode diagram of $G(s)$ with MATLAB. Explain why the phase-angle curve starts from 0° and approaches +180°.

B-8-7. Sketch the polar plots of the open-loop transfer function

$$G(s)H(s) = \frac{K(T_a s + 1)(T_b s + 1)}{s^2(Ts + 1)}$$

for the following two cases:

(a) $T_a > T > 0, \qquad T_b > T > 0$
(b) $T > T_a > 0, \qquad T > T_b > 0$

B-8-8. The pole–zero configurations of complex functions $F_1(s)$ and $F_2(s)$ are shown in Figures 8–133 (a) and (b), respectively. Assume that the closed contours in the s plane are those shown in Figures 8–133 (a) and (b). Sketch qualitatively the corresponding closed contours in the $F_1(s)$ plane and $F_2(s)$ plane.

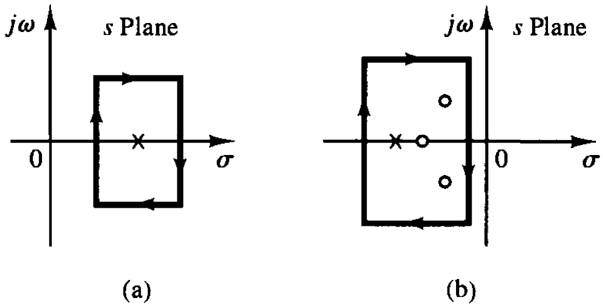

Figure 8–133 (a) s-Plane representation of complex function $F_1(s)$ and a closed contour; (b) s-Plane representation of complex function $F_2(s)$ and a closed contour.

B-8-9. Draw a Nyquist locus for the unity feedback control system with the open loop transfer function

$$G(s) = \frac{K(1 - s)}{s + 1}$$

Using the Nyquist stability criterion, determine the stability of the closed loop system.

B-8-10. A system with the open-loop transfer function

$$G(s)H(s) = \frac{K}{s^2(T_1 s + 1)}$$

is inherently unstable. This system can be stabilized by adding derivative control. Sketch the polar plots for the open-loop transfer function with and without derivative control.

B-8-11. Consider the closed-loop system with the following open-loop transfer function:

$$G(s)H(s) = \frac{10K(s + 0.5)}{s^2(s + 2)(s + 10)}$$

Plot both the direct and inverse polar plots of $G(s)H(s)$ with $K = 1$ and $K = 10$. Apply the Nyquist stability criterion to the plots and determine the stability of the system with these values of K.

B-8-12. Consider the closed-loop system whose open-loop transfer function is

$$G(s)H(s) = \frac{Ke^{-2s}}{s}$$

Find the maximum value of K for which the system is stable.

B-8-13. Draw a Nyquist plot of the following $G(s)$:

$$G(s) = \frac{1}{s(s^2 + 0.8s + 1)}$$

B-8-14. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the system.

B-8-15. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{s^2 + 2s + 1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the closed-loop system.

B-8-16. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

There are four individual Nyquist plots involved in this system. Draw two Nyquist plots for the input u_1 in one diagram and two Nyquist plots for the input u_2 in another diagram. Write a MATLAB program to obtain these two diagrams.

B-8-17. Referring to Problem B-8-16, it is desired to plot only $Y_1(j\omega)/U_1(j\omega)$ for $\omega > 0$. Write a MATLAB program to produce such a plot.

If it is desired to plot $Y_1(j\omega)/U_1(j\omega)$ for $-\infty < \omega < \infty$, what changes must be made in the MATLAB program?

B-8-18. Consider the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{as + 1}{s^2}$$

Determine the value of a so that the phase margin is 45°.

B-8-19. Consider the system shown in Figure 8–134. Draw a Bode diagram of the open-loop transfer function $G(s)$. Determine the phase margin and gain margin.

B-8-20. Consider the system shown in Figure 8–135. Draw a Bode diagram of the open-loop transfer function $G(s)$. Determine the phase margin and gain margin.

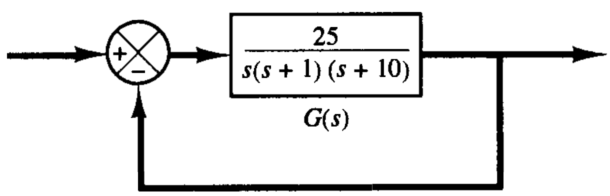

Figure 8–134 Control system.

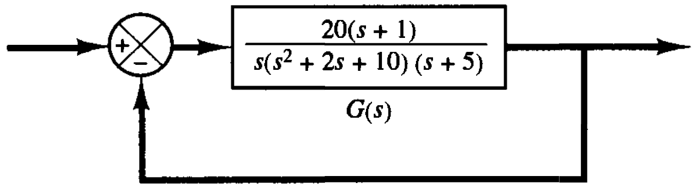

Figure 8–135 Control system.

B-8-21. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K}{s(s^2 + s + 4)}$$

Determine the value of the gain K such that the phase margin is 50°. What is the gain margin with this gain K?

B-8-22. Consider the system shown in Figure 8–136. Draw a Bode diagram of the open-loop transfer function and determine the value of the gain K such that the phase margin is 50°. What is the gain margin of this system with this gain K?

B-8-23. Consider a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s^2 + s + 0.5)}$$

Determine the value of the gain K such that the resonant peak magnitude in the frequency response is 2 dB, or $M_r = 2$ dB.

B-8-24. Figure 8–137 shows a block diagram of a process control system. Determine the range of gain K for stability.

B-8-25. Consider a closed-loop system whose open-loop transfer function is

$$G(s)H(s) = \frac{Ke^{-Ts}}{s(s + 1)}$$

Determine the maximum value of the gain K for stability as a function of dead time T.

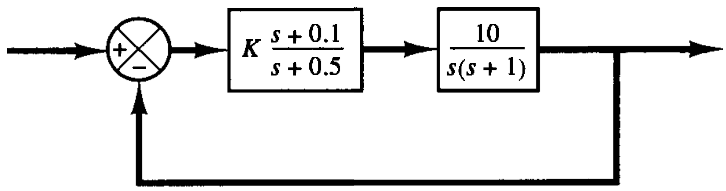

Figure 8–136 Control system.

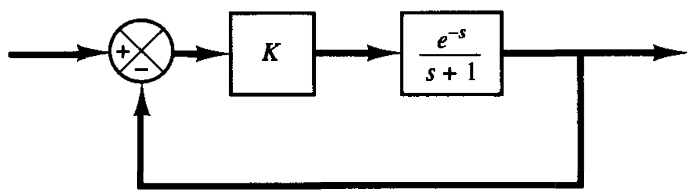

Figure 8–137 Process control system.

B-8-26. Sketch the polar plot of

$$G(s) = \frac{(Ts)^2 - 6(Ts) + 12}{(Ts)^2 + 6(Ts) + 12}$$

Show that, for the frequency range $0 < \omega T < 2\sqrt{3}$, this equation gives a good approximation to the transfer function of transport lag, e^{-Ts}.

B-8-27. Figure 8–138 shows a Bode diagram of a transfer function $G(s)$. Determine this transfer function.

B-8-28. The experimentally determined Bode diagram of a system $G(j\omega)$ is shown in Figure 8–139. Determine the transfer function $G(s)$.

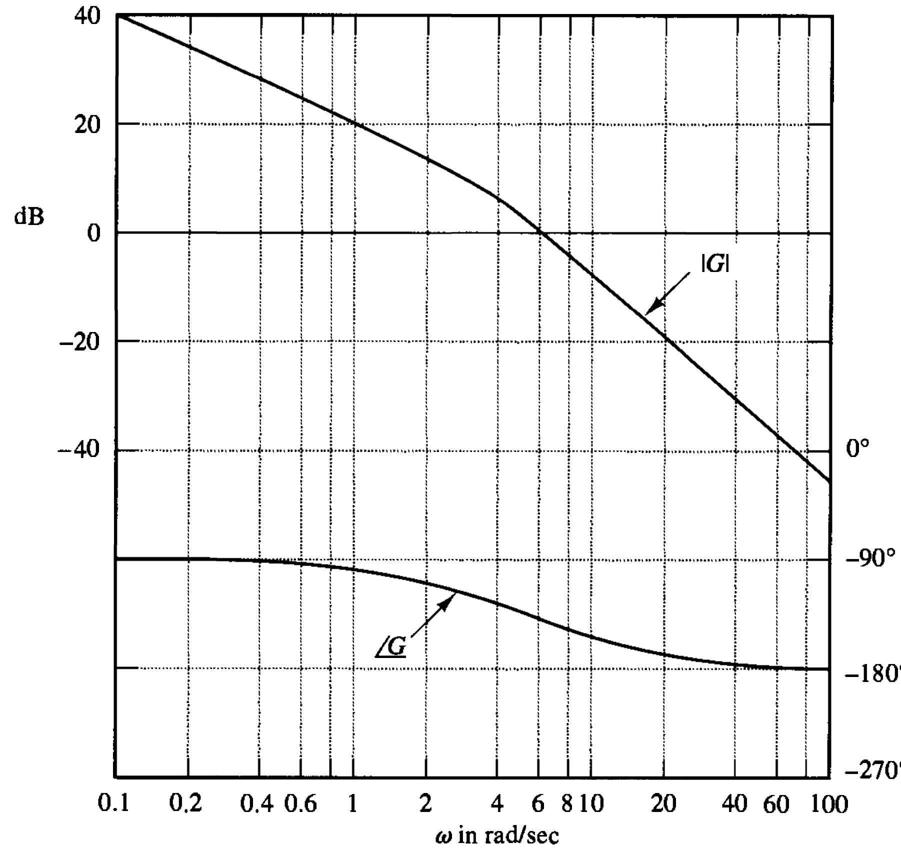

Figure 8–138 Bode diagram of a transfer function $G(s)$.

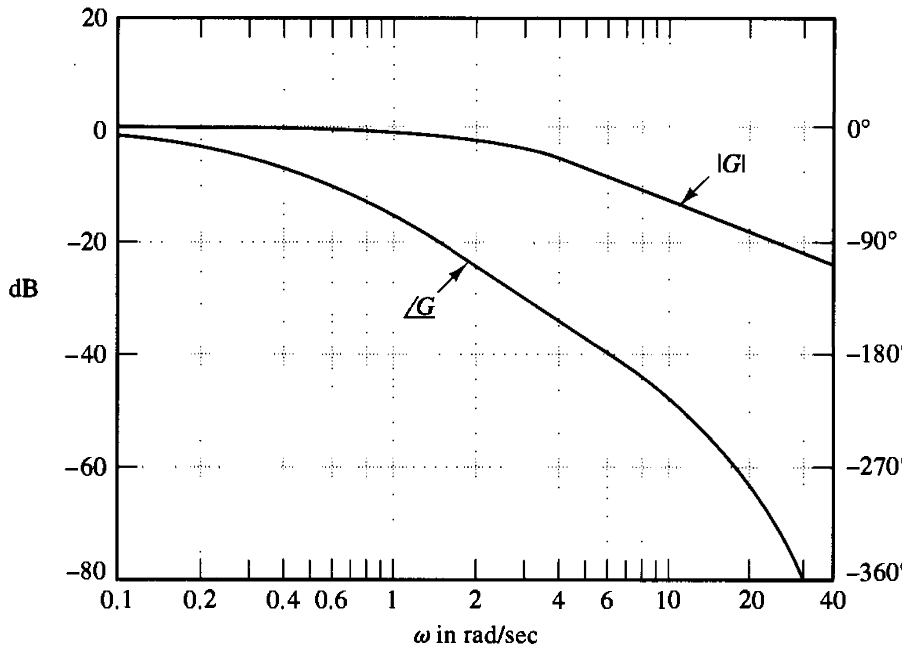

Figure 8–139 Experimentally determined Bode diagram of a system.