Hence

$$\int_0^\infty e(t)\,dt = \lim_{s\to 0} \frac{Q(s) - P(s)}{sQ(s)}$$
$$= \lim_{s\to 0} \frac{Q'(s) - P'(s)}{Q(s) + sQ'(s)}$$
$$= \lim_{s\to 0} [Q'(s) - P'(s)]$$

Since

$$\lim_{s\to 0} P'(s) = T_a + T_b + \cdots + T_m$$
$$\lim_{s\to 0} Q'(s) = T_1 + T_2 + \cdots + T_n$$

we have

$$\int_0^\infty e(t)\,dt = (T_1 + T_2 + \cdots + T_n) - (T_a + T_b + \cdots + T_m)$$

For a unit-step input $r(t)$, since

$$\int_0^\infty e(t)\,dt = \lim_{s\to 0} E(s) = \lim_{s\to 0} \frac{1}{1 + G(s)} R(s) = \lim_{s\to 0} \frac{1}{1 + G(s)} \frac{1}{s} = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_v}$$

we have

$$\frac{1}{\lim_{s\to 0} sG(s)} = (T_1 + T_2 + \cdots + T_n) - (T_a + T_b + \cdots + T_m)$$

Note that zeros in the left half-plane (that is, positive $T_a, T_b, \ldots, T_m$) will improve K_v. Poles close to the origin cause low velocity-error constants unless there are zeros nearby.

PROBLEMS

B-5-1. If the feedforward path of a control system contains at least one integrating element, then the output continues to change as long as an error is present. The output stops when the error is precisely zero. If an external disturbance enters the system, it is desirable to have an integrating element between the error-measuring element and the point where the disturbance enters so that the effect of the external disturbance may be made zero at steady state.

Show that, if the disturbance is a ramp function, then the steady-state error due to this ramp disturbance may be eliminated only if two integrators precede the point where the disturbance enters.

B-5-2. Consider industrial automatic controllers whose control actions are proportional, integral, proportional-plus-integral, proportional-plus-derivative, and proportional-plus-integral-plus-derivative. The transfer functions of these controllers can be given, respectively, by

$$\frac{U(s)}{E(s)} = K_p$$
$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$
$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$
$$\frac{U(s)}{E(s)} = K_p(1 + T_d s)$$
$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

where $U(s)$ is the Laplace transform of $u(t)$, the controller output, and $E(s)$ the Laplace transform of $e(t)$, the actuating error signal. Sketch $u(t)$ versus t curves for each of the five types of controllers when the actuating error signal is

(a) $e(t)$ = unit-step function

(b) $e(t)$ = unit-ramp function

In sketching curves, assume that the numerical values of K_p, K_i, T_i, and T_d are given as

$$K_p = \text{proportional gain} = 4$$

$$K_i = \text{integral gain} = 2$$

$$T_i = \text{integral time} = 2 \text{ sec}$$

$$T_d = \text{derivative time} = 0.8 \text{ sec}$$

B-5-3. Consider a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(Js + B)}$$

Discuss the effects that varying the values of K and B has on the steady-state error in unit-ramp response. Sketch typical unit-ramp response curves for a small value, medium value, and large value of K.

B-5-4. Figure 5–82 shows three systems. System I is a positional servo system. System II is a positional servo system with PD control action. System III is a positional servo system with velocity feedback. Compare the unit-step, unit-impulse, and unit-ramp responses of the three systems. Which system is best with respect to the speed of response and maximum overshoot in the step response?

B-5-5. Consider the position control system shown in Figure 5–83. Write a MATLAB program to obtain a unit-step response and a unit-ramp response of the system. Plot curves $x_1(t)$ versus t, $x_2(t)$ versus t, $x_3(t)$ versus t, and $e(t)$ versus t [where $e(t) = r(t) - x_1(t)$] for both the unit-step response and the unit-ramp response.

B-5-6. Determine the range of K for stability of a unity-feedback control system whose open-loop transfer function is

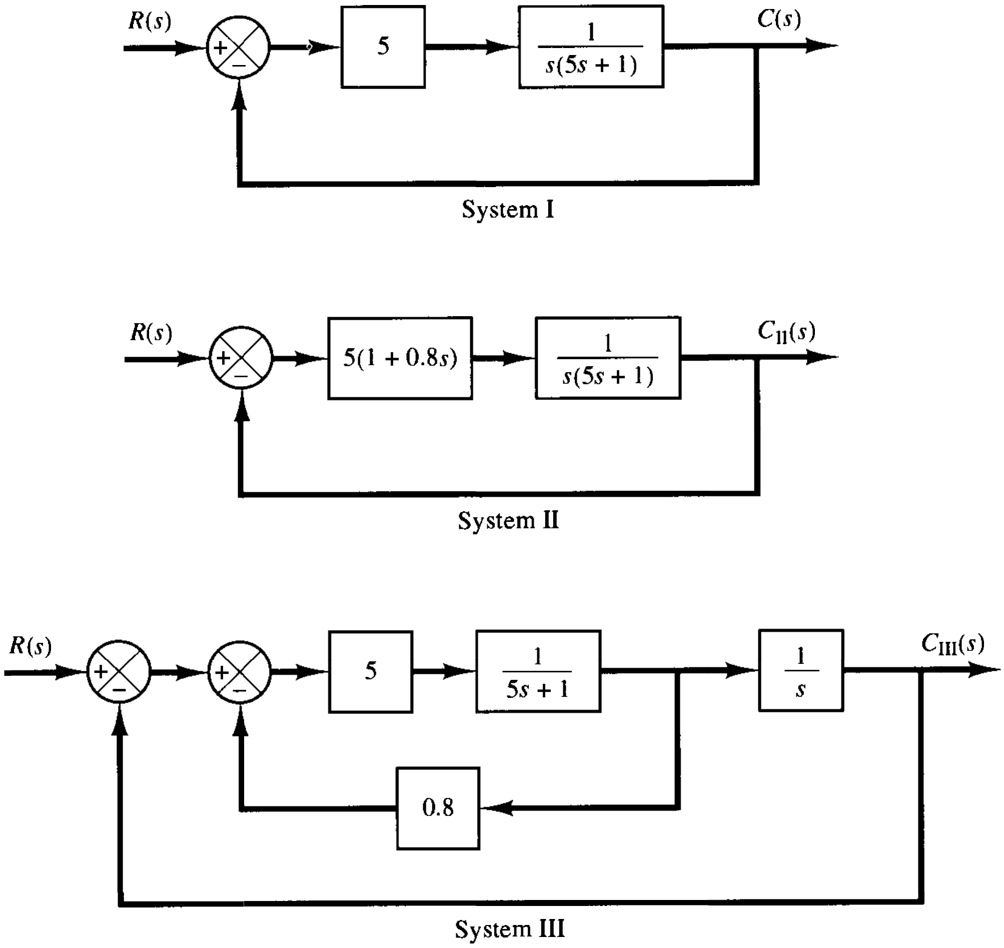

Figure 5–82
(a) Positional servo system; (b) positional servo system with PD control action; (c) positional servo system with velocity feedback.

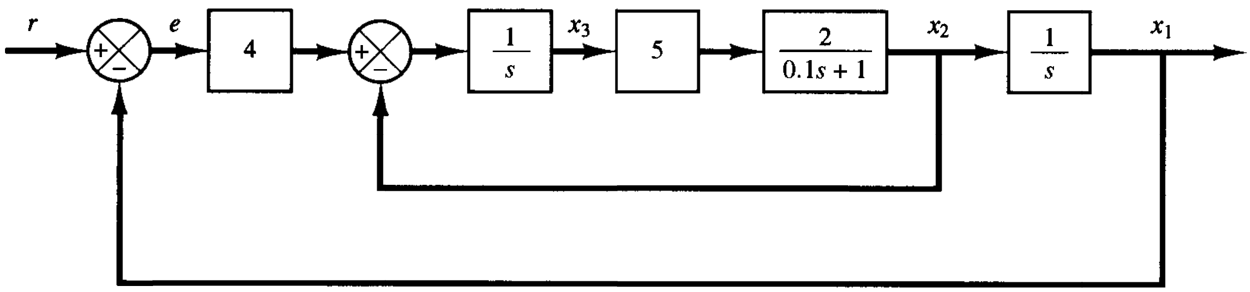

Figure 5–83 Position control system.

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

B-5-7. Consider the unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{10}{s(s-1)(2s+3)}$$

Is this system stable?

B-5-8. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -b_3 & 0 & 1 \\ 0 & -b_2 & -b_1 \end{bmatrix}$$

($\mathbf{A}$ is called Schwarz matrix.) Show that the first column of the Routh's array of the characteristic equation $|s\mathbf{I} - \mathbf{A}| = 0$ consists of 1, b_1, b_2, and $b_1 b_3$.

B-5-9. Consider the pneumatic system shown in Figure 5–84. Obtain the transfer function $X(s)/P_i(s)$.

B-5-10. Figure 5–85 shows a pneumatic controller. What kind of control action does this controller produce? Derive the transfer function $P_c(s)/E(s)$.

B-5-11. Consider the pneumatic controller shown in Figure 5–86. Assuming that the pneumatic relay has the characteristics that $p_c = Kp_b$ (where $K > 0$), determine the control action of this controller. The input to the controller is e and the output is p_c.

B-5-12. Figure 5–87 shows a pneumatic controller. The signal e is the input and the change in the control pressure p_c is the output. Obtain the transfer function $P_c(s)/E(s)$. Assume that the pneumatic relay has the characteristics that $p_c = Kp_b$, where $K > 0$.

B-5-13. Consider the pneumatic controller shown in Figure 5–88. What control action does this controller produce? Assume that the pneumatic relay has the characteristics that $p_c = Kp_b$, where $K > 0$.

B-5-14. Figure 5–89 shows an electric-pneumatic transducer. Show that the change in the output pressure is proportional to the change in the input current.

B-5-15. Figure 5–90 shows a flapper valve. It is placed between two opposing nozzles. If the flapper is moved slightly to the right, the pressure unbalance occurs in the nozzles and the power piston moves to the left, and vice versa. Such a device is frequently used in hydraulic servos as the first-stage valve in two-stage servovalves. This usage occurs because considerable force may be needed to stroke larger spool valves that result from the steady-state flow force. To reduce or compensate this force, two-stage valve configuration is often employed; a flapper valve or jet pipe is used as the first-stage valve to provide a necessary force to stroke the second-stage spool valve.

Figure 5–91 shows a schematic diagram of a hydraulic servomotor in which the error signal is amplified in two stages using a jet pipe and a pilot valve. Draw a block diagram of the system of Figure 5–91 and then find the trans-

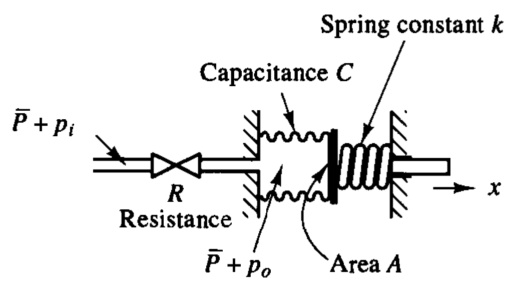

Figure 5–84 Pneumatic system.