

Bode Plots

- A polar plot gives information in one plot but is harder to use for design purposes.

Note that $G(j\omega) = |G(j\omega)|e^{j\phi(\omega)}$.

- Bode plots - plot of the magnitude $|G(j\omega)|$ vs. ω and the phase $\angle G(j\omega)$ vs. ω .

- The plots are on a ω -axis scale. The magnitude is plotted using the logarithmic gain.

Bode Plots

- Simple example. filter.

$$G(s) = \frac{1}{RCs + 1} \Rightarrow G(j\omega) = \frac{1}{j\omega RC + 1}$$

$$|G(j\omega)| = \sqrt{\frac{1}{1 + (\omega RC)^2}}, \quad \phi(\omega) = -\tan^{-1}(\omega RC)$$

- gain.

$$20 \log |G(j\omega)| = 20 \log \sqrt{\frac{1}{1 + (\omega RC)^2}}$$

Bode Plots

- Simplify.

$$20 \log |G(j\omega)| = -10 \log[1 + (\omega RC)^2]$$

- Let $\tau = RC$.

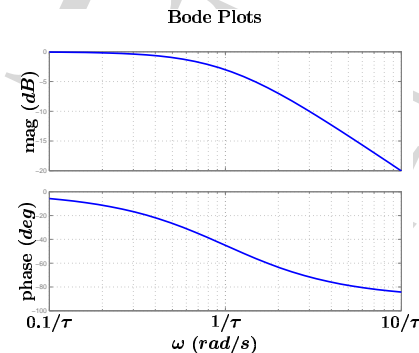
$$\omega \ll \frac{1}{\tau} : 20 \log |G(j\omega)| \approx -10 \log(1) = 0dB$$

$$\omega = \frac{1}{\tau} : 20 \log |G(j\omega)| = -10 \log(2) = -3.01dB$$

$$\omega \gg \frac{1}{\tau} : 20 \log |G(j\omega)| \approx -20 \log(\omega\tau)$$

Bode Plots

- Define $\omega_c = \frac{1}{\tau}$ as the frequency.



Decade : of two frequencies is 10.

Example. The frequencies $\omega_1 = 10 \text{ rad/s}$ and $\omega_2 = 100 \text{ rad/s}$ are one decade apart.

What is an ve?

Bode Plots

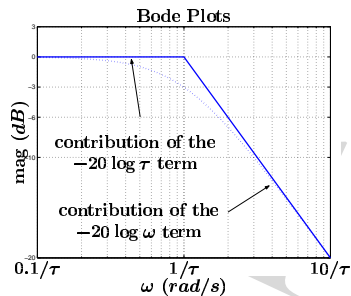
- Logarithmic scale.
 $\Rightarrow \log \omega$ terms are plotted as linear curves on the graph.

For high frequencies ω ,

$$20 \log |G(j\omega)| \approx -20 \log(\omega\tau) = -20 \log \tau - 20 \log \omega$$

$-20 \log \tau$ term.
 contributes a constant offset on the graph

$-20 \log \omega$ term.
 contributes a linear curve on the graph.



Advantage of Using Bode Plots

- The new magnitude plot is the graph of $20 \log |\tilde{G}(j\omega)|$.

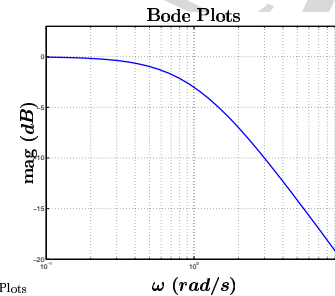
$$\begin{aligned} 20 \log |\tilde{G}(j\omega)| &= 20 \log \left| G(j\omega) \cdot \frac{1}{j\omega - p} \right| \\ &= 20 \log |G(j\omega)| + 20 \log \left| \frac{1}{j\omega - p} \right| \end{aligned}$$

- Since we have the magnitude plot of $G(s)$, we only need to add the magnitude plot of pole to the original plot to get the magnitude plot for $\tilde{G}(s)$.

Advantage of Using Bode Plots

- Since the magnitude plot is $20 \log |G(j\omega)|$, multiplication operation on the transfer function translates to an addition operation on the graph.

Example. Consider the transfer function $G(s)$ with the following Bode plot.



Let us say you want to add a pole to $G(s)$.

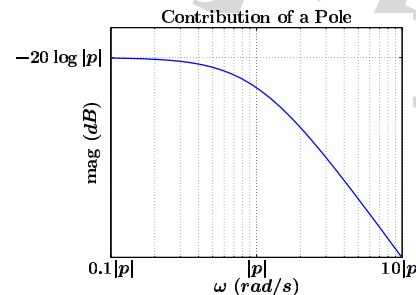
$$\tilde{G}(s) = G(s) \cdot \frac{1}{s - p}$$

What is the effect of the pole on the magnitude plot?

Advantage of Using Bode Plots

- Contribution of the pole.

$$\frac{1}{s - p} = \frac{1}{p} \cdot \frac{1}{\frac{s}{p} + 1} \xrightarrow{s=j\omega} -\frac{1}{p} \cdot \frac{1}{\frac{j\omega}{-p} + 1}$$



Bode plot is similar to the Bode plot of an RC filter.

Corner frequency is at

$$\omega = |p|$$

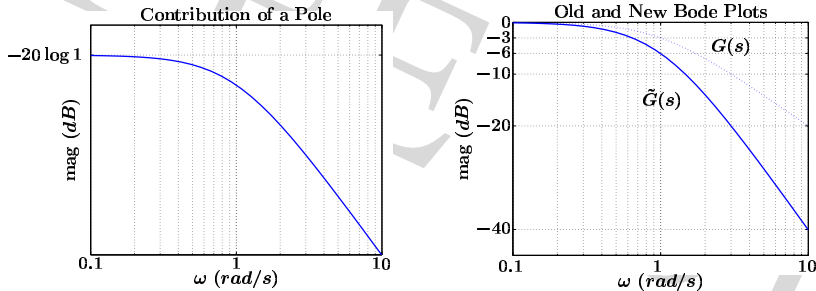
For $\omega \ll |p|$,

$$\text{magnitude} = -20 \log |p|$$

Advantage of Using Bode Plots

- Let us say we want to add at $\{p = -1\}$ to $G(s)$.

Adding the contribution of the pole to the original TF, we get the magnitude plot for $\tilde{G}(s)$.



Advantage of Using Bode Plots

- Given a transfer function,

$$G(s) = k(s - z_1) \dots (s - z_m) \cdot \frac{1}{(s - p_1)} \dots \frac{1}{(s - p_n)}$$

To determine the magnitude plot of $G(s)$, add the appropriate magnitude plots for k , zero terms and pole terms.

- What about the phase plot?

The phase plot can also be determined by decomposing the transfer function into terms with zeros and poles.

Advantage of Using Bode Plots

- To determine the effect of additional poles and zeros, and additional gain on the system, modify the Bode plot. Can easily be accomplished by adding the appropriate contribution to the magnitude plot.

- Useful to decompose the transfer function into standard factors.
 - pure gain.
 - poles / zeros at the origin.
 - poles / zeros on the real axis.
 - complex conjugate poles / zeros.

Advantage of Using Bode Plots

- Since the pure gain k has no angle contribution,

$$\text{angle of } G(s) = \begin{cases} \sum \text{angle contribution of zero terms} \\ - \sum \text{angle contribution of pole terms} \end{cases}$$

- Thus, determine the phase plot of a transfer function by plotting the angle contribution of each of the zero / pole terms and adding / subtracting the plots appropriately.

To determine the effect of additional zeros / poles on the phase of a transfer function, add / subtract the appropriate phase plot(s) to / from the original phase plot.

Bode Plot for Pure Gain

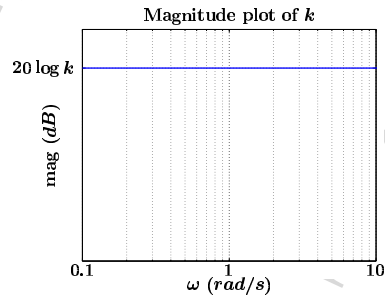
- Pure gain.

$$k \xrightarrow{s=j\omega} k$$

- Magnitude plot.

$$20 \log |k| = \text{constant}$$

⇒ magnitude plot is a line intersecting the magnitude axis at $20 \log k$ dB.

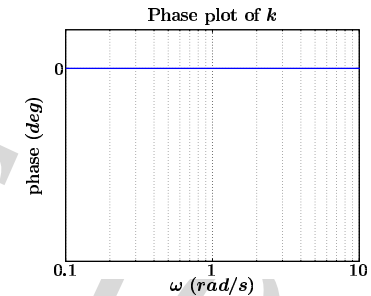


Bode Plot for Pure Gain

- Phase plot.

$$\phi(\omega) = \angle k = 0$$

⇒ no effect on the phase plot.



- Additional $20 \log k$ to a transfer function.
⇒ offset the original magnitude plot by $20 \log k$ dB.
⇒ same (original) phase plot.

Bode Plot for Poles / Zeros at the Origin

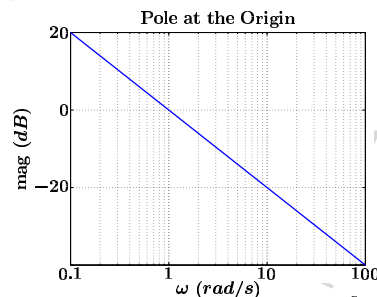
- Pole at the origin.

$$\frac{1}{s} \xrightarrow{s=j\omega} \frac{1}{j\omega}$$

- Magnitude plot.

$$20 \log \left| \frac{1}{j\omega} \right| = -20 \log \omega$$

⇒ slope of the curve is -20 dB/dec.

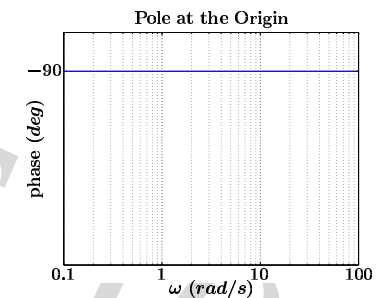


Bode Plot for Poles / Zeros at the Origin

- Phase plot.

$$\phi(\omega) = \angle \left[\frac{1}{j\omega} \right] = -90^\circ$$

⇒ phase plot is a line intersecting the phase axis at -90° .



- Additional pole at the origin to a transfer function.
⇒ add the $-20 \log \omega$ line to the original magnitude plot.
⇒ offset the phase plot by -90° .

Bode Plot for Poles / Zeros at the Origin

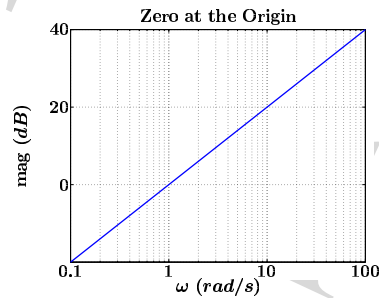
- Zero at the origin.

$$s \xrightarrow{s=j\omega} j\omega$$

- Magnitude plot.

$$20 \log |j\omega| = 20 \log \omega$$

⇒ slope of the magnitude is 20 dB/dec.

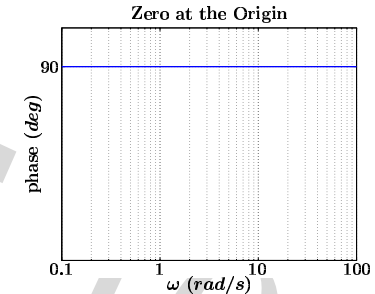


Bode Plot for Poles / Zeros at the Origin

- Phase plot.

$$\phi(\omega) = \angle j\omega = 90^\circ$$

⇒ phase plot is a horizontal line at 90°.



- Additional zero at the origin to a transfer function.
⇒ add the 20 log ω line to the original magnitude plot.
⇒ offset the phase plot by 90°.

Bode Plot for Poles / Zeros at the Origin

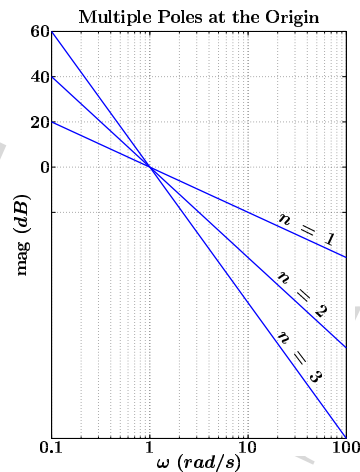
- Multiple poles at the origin.

$$20 \log \left| \frac{1}{(j\omega)^n} \right| = -20n \log \omega$$

⇒ slope of the magnitude curve decreases by 20 dB/dec for every additional pole at the origin.

$$\phi(\omega) = \angle \left[\frac{1}{(j\omega)^n} \right] = -90^\circ n$$

⇒ additional -90° phase shift for every pole at the origin.



Bode Plot for Poles / Zeros on the Real Axis

- Pole on the real axis.

$$\frac{1}{s-p} = -\frac{1}{p} \cdot \frac{1}{\frac{s}{-p} + 1}$$

- Treat the factor $-\frac{1}{p}$ separately as an additional gain and an additional phase shift if $p > 0$.

Let $\tau = \frac{1}{|-p|}$, then examine the Bode plot of

$$\frac{1}{\tau s + 1} \xrightarrow{s=j\omega} \frac{1}{j\omega\tau + 1}$$

Bode Plot for Poles / Zeros on the Real Axis

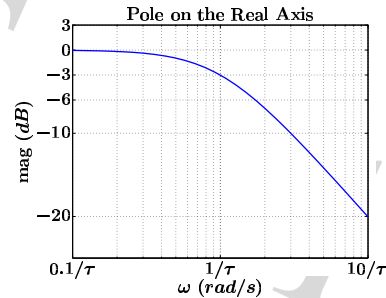
- Look at the

$$\left| \frac{1}{j\omega\tau + 1} \right| = \sqrt{\frac{1}{1 + (\omega\tau)^2}}$$

Magnitude plot.

$$20 \log \left| \frac{1}{j\omega\tau + 1} \right| = -10 \log(1 + \omega^2\tau^2)$$

$$\Rightarrow \omega_c = \frac{1}{\tau} = |p|$$

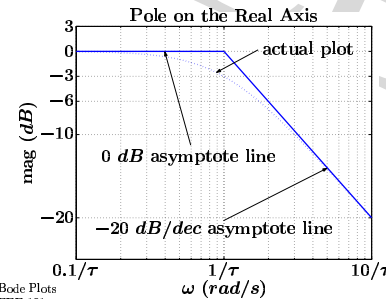


Bode Plot for Poles / Zeros on the Real Axis

- Look at the

$\omega \ll \frac{1}{\tau}$: asymptote line is $-10 \log(1) = 0dB$ line.

$\omega \gg \frac{1}{\tau}$: asymptote line is $-20 \log(\omega\tau)$ line.



Intersection of asymptotes.

$$-10 \log(1) = -20 \log(\omega\tau)$$

$$\Rightarrow \omega = \frac{1}{\tau}$$

\Rightarrow corner

Bode Plot for Poles / Zeros on the Real Axis

- Zero on the real axis.

$$s - z = -z \cdot \left[\frac{s}{-z} + 1 \right]$$

- Treat the factor $-z$ as an additional gain and an additional 180° phase shift if $z > 0$.

- Let $\tau = \frac{1}{|-z|}$, then the Bode plot of

$$\tau s + 1 \xrightarrow{s=j\omega} j\omega\tau + 1$$

Bode Plot for Poles / Zeros on the Real Axis

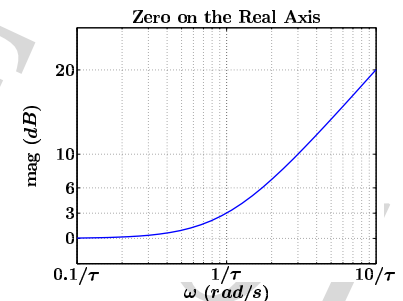
- Magnitude.

$$|j\omega\tau + 1| = \sqrt{1 + (\omega\tau)^2}$$

Magnitude plot.

$$20 \log |j\omega\tau + 1| = 10 \log(1 + \omega^2\tau^2)$$

$$\Rightarrow \omega_c = \frac{1}{\tau} = |z|$$

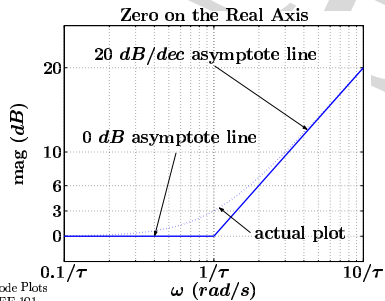


Bode Plot for Poles / Zeros on the Real Axis

- Look at asymptotes.

$\omega \ll \frac{1}{\tau}$: asymptote line is $10 \log(1) = 0 \text{ dB}$ line.

$\omega \gg \frac{1}{\tau}$: asymptote line is $20 \log(\omega\tau)$ line.



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Intersection of asymptotes.

$$10 \log(1) = 20 \log(\omega\tau)$$

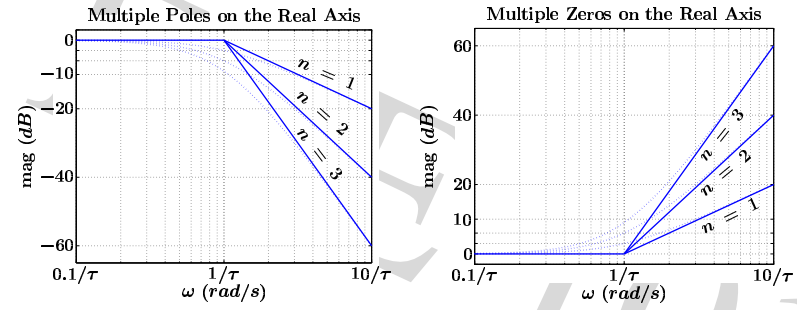
$$\Rightarrow \omega = \frac{1}{\tau}$$

\Rightarrow frequency

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Bode Plot for Poles / Zeros on the Real Axis

- Multiple on the real axis.



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$$20 \log \left| \frac{1}{(j\omega\tau + 1)^n} \right| = -10n \log(1 + \omega^2\tau^2)$$

$$20 \log |j\omega\tau + 1|^n = 10n \log(1 + \omega^2\tau^2)$$

Remarks on Bode Plots

- Exercise / Homework.

Bode plots for complex conjugate poles / zeros.

- Asymptotic bode plots are more to use for approximations.
 - \Rightarrow easier to add lines than individual points.
 - \Rightarrow draw Bode plots first, and then fill-in the 'exact' plot by marking off the $\pm 3 \text{ dB}$ points.

- Bode plots for poles and zeros only differ by a negative sign. Symmetrical about the axis.

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