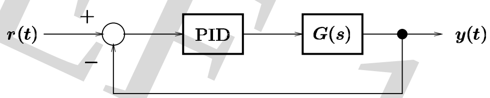
PID Controller Lecture

- Root locus helps us visualize what happens when we vary the , add poles, and add zeros.

  Is there a common approach to modifying the behavior?



- How are systems usually designed? Dominant poles?

- Study proportional controller design.
  Then, look at proportional derivative controller .

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P / PD / PID Controller Design

- P - proportional (for error tracking)
  D - derivative (for faster controller response)
  I - integral (to drive the error to zero)

- Performance specifications are given based on the step response of a system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ : damping factor
ω_n : natural frequency

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P / PD / PID Controller Design

- Step response.

$$Y(s) = G(s) \cdot \frac{1}{s}$$

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \beta t - \theta), \; t \geq 0$$

where $\theta = \cos^{-1}\zeta$, for $0 < \zeta < 1$

- Specify performance .
  $\Rightarrow$ determine appropriate ζ and ω_n.

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P / PD / PID Controller Design

- Some performance specifications.
  - T_s : time - time for the system output to settle within a certain % of the steady-state value.

    for $\pm 2\%$: $e^{-\zeta\omega_n T_s} < 0.02 \Rightarrow T_s \approx \frac{4}{\zeta\omega_n}$
  - T_p : time to .

$$T_p = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}}$$

  - M_p : peak overshoot.

$$M_p = 100 e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}}$$

  - rise time, T_r and delay time, T_d (see text).

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P / PD / PID Controller Design

- Giving performance specifications will what are the required ζ and ω_n, and the poles of the second-order system

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

- Poles determine the response of a closed-loop system.

To design a control system which at s the performance specifications, set

$$\begin{matrix}\text{dominant poles of the}\\ \text{closed-loop system}\end{matrix} = s_{1,2}$$

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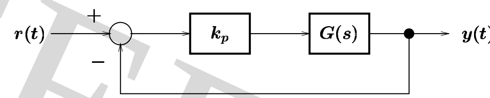
Proportional Controller

- Typical proportional controller.



- Design problem. Determine k_p such that the performance specifications are met.
- Approach. Use root locus to find k_p such that the response due to the roots (poles) meet the performance specifications.

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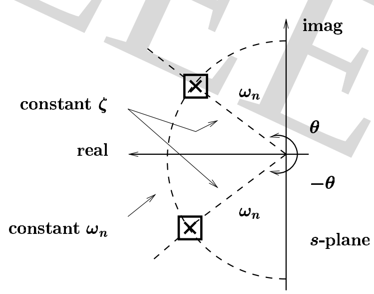
Proportional Controller

- Second-order system.

ple form : $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$

polar form : $s_{1,2} = \omega_n\angle \pm \theta$



$$\theta = \tan^{-1}\left[\frac{\sqrt{1 - \zeta^2}}{-\zeta}\right]$$

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Proportional Controller

- Example. Design a proportional controller for the following system

$$G(s) = \frac{s + 0.3}{1000s^2 + 80s + 1.5}$$

such that the following specifications are satisfied.

$$T_s < 18.5\ s \text{ and } T_p < 10.9\ s$$

- Using the equations for T_s and T_p, and for the parameters ζ and ω_n gives

$$\zeta > 0.6 \text{ and } \omega_n > 0.36\ rad/s$$

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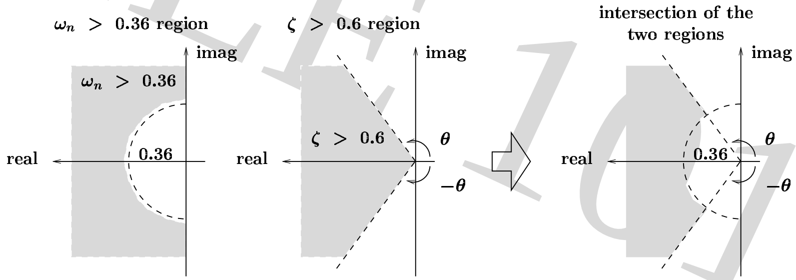
Proportional Controller

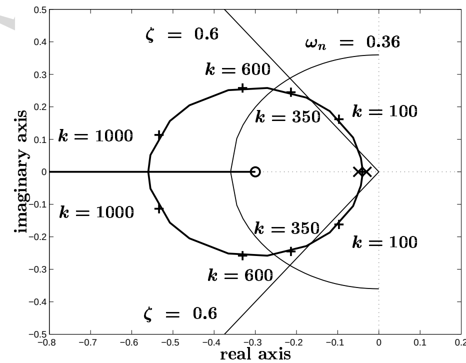
• regions to consider.

$$\zeta = 0.6 \Rightarrow \theta = \tan^{-1}\left[\frac{\sqrt{1 - 0.6^2}}{-0.6}\right] = 126.87^o$$



Proportional Controller

• Use the root locus to find the gain k_p such that the dominant poles fall inside the desired .



From the root locus, we can see that for about $k > 600$, the roots are inside the desired region.

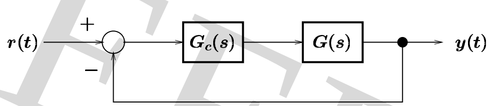
Thus we can choose

$$k_p = 600$$

for our proportional controller design.

Proportional Derivative Controller

• Typical Controller.



$$G_c(s) = k_p + k_d s$$

• Design problem. Determine k_p and k_d such that the performance specifications are met.

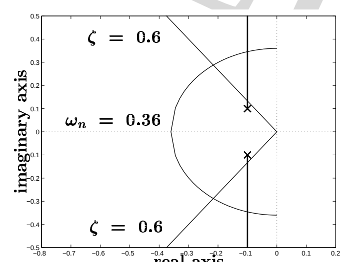
• Approach. Use root to find k_p and k_d such that the response due to the dominant roots meet the performance specifications.

Proportional Derivative Controller

• Why use a PD controller?

Consider the example but with the plant transfer function.

$$G(s) = \frac{1}{s^2 + 0.2s + 0.02}$$



From the performance specifications we still have

$\zeta > 0.6$ and $\omega_n > 0.36\ rad/s$

The root locus will never enter the desired region for any value of k.

Proportional Derivative Controller

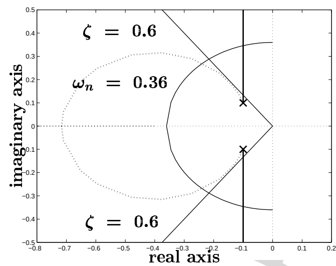
- Thus, if we try to use a P controller in our design, we will never find a k_p to satisfy the requirements.

  Proportional control is not to achieve our desired performance specifications.

  What can we do?

  If we can somehow bend the root locus to the left.

  We can introduce a zero somewhere on the negative real axis.



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Proportional Derivative Controller

- Solution. Use a PD .

- The PD controller introduces a zero to the forward TF.

$$G_c(s) = k_p + k_d s = k_d\left(s + \frac{k_p}{k_d}\right)$$

Let $z_c = -\dfrac{k_p}{k_d}$.

- The zero z_c changes the root locus of the system.

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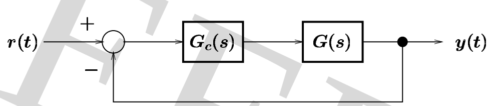
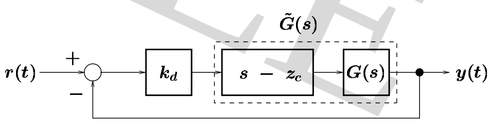
Proportional Derivative Controller

- PD controller, a look.



$$G_c(s) = k_p + k_d s$$



$$\tilde{G}(s) = (s - z_c)G(s)$$

$$z_c = -\frac{k_p}{k_d}$$

- We can use the root locus for the design based on the equation : $1 + k_d\tilde{G}(s) = 0$.

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Proportional Derivative Controller

- Example. Design an appropriate controller for the following system

$$G(s) = \frac{4}{s^2 + 0.2s + 0.02}$$

such that the following performance specifications are satisfied.

$$T_s < 18.5\ s \text{ and } T_p < 10.9\ s$$

- Using the equations T_s and T_p, and solving for the parameters ζ and ω_n gives

$$\zeta > 0.6 \text{ and } \omega_n > 0.36\ rad/s$$

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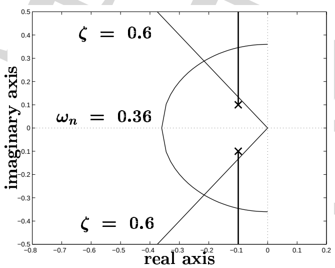
Proportional Derivative Controller

- Looking at the root locus and the region,



If we try to use a P controller in our design, we will never find a k_p to satisfy the requirements.

We can not satisfy the control requirements with a proportional controller.

- Try using a controller.

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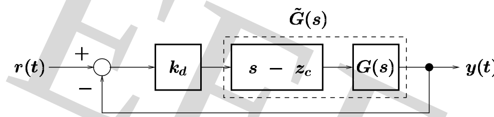
Proportional Derivative Controller

- Design the PD controller with $G_c(s) = k_p + k_d s$.



$$\tilde{G}(s) = (s - z_c)G(s)$$

$$z_c = -\frac{k_p}{k_d}$$

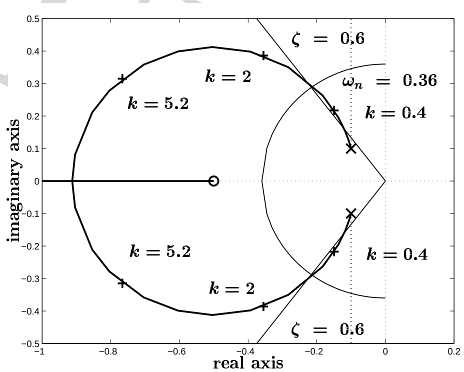
- We need to bend the root locus to the left.

From the simple rules, we can that adding a pole anywhere to the left of $s = -0.1$ will work.

$\Rightarrow$ choose $z_c = -0.5$.

Proportional Derivative Controller

- Root locus of $\tilde{G}(s)$.



From the root locus, we can see that for about $k > 0.5$, the roots are inside the desired region.

Thus we can choose

$$k_d = 2$$

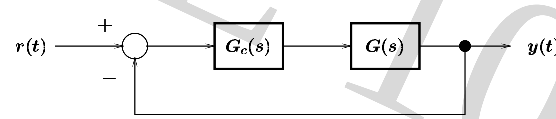
for our PD design.

Proportional Derivative Controller

- Then computing k_p using $z_c = -0.5$ and $k_d = 2$,

$$z_c = -\frac{k_p}{k_d} \Rightarrow k_p = 1$$

- Thus, our proportional derivative controller will be



$$G_c(s) = 1 + 2s$$

Proportional Integral Derivative Controller

- Recall that adding an integrator increases system type.
  ⇒ drives the steady-state error to zero.
- However, the also introduces a pole at the origin of the s-plane.
  ⇒ may lead to marginal stability.
- Use a PID (or PI) controller to marginal stability problem but still get the benefit of having the integrator in the system.

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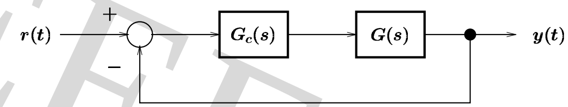
Proportional Integral Derivative Controller

- Typical PID controller (PI controller for $k_d = 0$).



$$G_c(s) = k_p + k_d s + \frac{k_i}{s}$$

- at pole at the origin and two additional zeros.

$$G_c(s) = k_p + k_d s + \frac{k_i}{s} = \frac{k_p s + k_d s^2 + k_i}{s}$$

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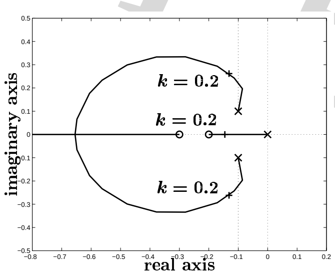
Proportional Integral Derivative Controller

- Design using the PID controller.
  ⇒ choose one of the zeros of $G_c(s)$ to pull the locus from the pole at the origin towards the LHP.
  ⇒ choose the other zero of $G_c(s)$ to the locus of the original system as with a PD controller.



The original system has two poles at

$$p_{1,2} = -0.1 \pm j0.1$$

The zeros of $G_c(s)$ are located at

$$z_{c1,c2} = -0.2, -0.3$$

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Summary

- Design systems by putting the dominant poles in the right locations.
- PID controller is the most common type of used in industry; the controller is suitable for most process control requirements.

  For most processes, PI or PD control is sufficient.
- The PID controller can be easily by setting the constants k_p, k_d and k_i by using standard techniques or by trial and error.

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