Outline of Today's Lecture

- What are determinants, and how do they help us to see whether a system is stable or not.
- What is the Routh-Hurwitz criterion? What is an R-H table?
- Why do you want to use Routh-Hurwitz stability test? Why not just find the poles and decide from there?
- Some examples and on using R-H stability test.

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Routh-Hurwitz Stability Test

- Why study the Routh-Hurwitz test?

  The Routh-Hurwitz test is a method for whether or not a system is stable based upon the coefficients in the system's characteristic equation.
- Quick check for stability given the polynomial.
- Based on the concept of Hurwitz determinants.

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Routh-Hurwitz Stability Test

- Theorem.
  All roots lie in the if and only if Hurwitz determinants are positive.

$$D(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0$$

$$D_1 = a_1$$

$$D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}$$

$$\vdots \quad \vdots$$

$$D_n = \ldots$$

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Routh-Hurwitz Stability Test

- If we already can deduce from Hurwitz determinants, what did Routh do?

  Routh simplified the Hurwitz determinants into a tabular method.
- Useful for determining stability limits, i.e., what gain k makes a control stable.
- Particularly useful for higher-order systems because it does not require the polynomial in the transfer function to be factored.

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The Method

- Write the characteristic equation in the following form.

$$a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0$$

- Important note before using R-H test.

  If any of the coefficients are zero or negative and at least one of the coefficients is positive, there is a there are roots that are imaginary or that have positive real parts.

  Then, the system is stable or unstable.

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The Method

- Routh-Hurwitz (R-H) .

s^n :	a_0	a_2	$a_4 \ldots$
s^{n-1} :	a_1	a_3	$a_5 \ldots$
s^{n-2} :	$b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$	$b_2 = \dfrac{a_1 a_4 - a_0 a_5}{a_1}$	$\ldots\ldots$
s^{n-3} :	$c_1 = \dfrac{b_1 a_3 - a_1 b_2}{b_1}$	$\ldots$	$\ldots$
$\vdots$	$\vdots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	
s^1 :	$d_1 = \ldots$	$d_2 = \ldots$	
s^0 :	$e_1 = \ldots$		

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The Method

- The Routh- stability criterion states that the number of roots with positive real parts is equal to the number of sign changes of the coefficients in the first column of the table.

- Note that the exact values are not required for the coefficients; only the sign matters.

- Used to be harder to find than apply R-H test.

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The Method

- R-H test may be used to determine the regions of stability, i.e., what range of feedback parameters will the system stable.

- Root locus provides the same information in a graphical manner.

- Advances in computing have reduced the usage of R-H test.

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Examples

- **Simple example.**
  Given a system with characteristic equation

$$a_0 s^2 + a_1 s + a_2 = 0$$

  Determine which values of the coefficients will make the system stable and which will make the system .

- **Arranged in R-H table form.**

$$\begin{array}{lll} s^2 : & a_0 & a_2 \\ s^1 : & a_1 & \\ s^0 : & \boxed{\dfrac{a_1 a_2 - a_0 \cdot 0}{a_1}} & \end{array}$$

Examples

- **Simplified.**

$$\begin{array}{l} s^2 : a_0 \; a_2 \\ s^1 : a_1 \\ s^0 : a_2 \end{array}$$

- **According to R-H, for , there should be no sign change in the first column.**

- **Therefore, for , $a_i > 0$ for all $i = 0, 1, 2$.**

Examples

- **Another example.**
  Consider the system with characteristic equation

$$s^3 + s^2 + 2s + 24 = 0$$

- **R-H table format.**

$$\begin{array}{lll} s^3 : & 1 & 2 \\ s^2 : & 1 & 24 \\ s^1 : & \boxed{\dfrac{1 \cdot 2 - 1 \cdot 24}{1} = -22} & \\ s^0 : & \boxed{\dfrac{-22 \cdot 24 - 1 \cdot 0}{-22} = 24} & \end{array}$$

Examples

- **Since there is at least one sign in the first row of the R-H table, the system is unstable.**

- **Check with .**

```
>> roots([1 1 2 24])

ans =

  -3.0000
   1.0000 + 2.6458i
   1.0000 - 2.6458i
```

⇒ It has two roots in the right half-plane.

⇒ The system is unstable.

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- Illustrative example.
  What if a appears in the first column?

  Consider the system with characteristic equation

$$s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

- Constructing the table,

s^5 :	1	2	11
s^4 :	2	4	10
s^3 :	$\dfrac{2\cdot 2 - 1\cdot 4}{2} = 0$	$\dfrac{2\cdot 11 - 1\cdot 10}{2} = 6$	

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- A zero appears in the first column.
  We will have a problem constructing the succeeding of R-H table (division by zero).

- Recall that we are interested only in the sign of the coefficients.

- Workaround is to replace the 0 with a , positive number, ϵ.

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- Continue constructing the R-H table.

s^5 :	1	2	11
s^4 :	2	4	10
s^3 :	ϵ	6	
s^2 :	$\dfrac{\epsilon\cdot 4 - 2\cdot 6}{\epsilon} = \dfrac{-12}{\epsilon}$	$\dfrac{\epsilon\cdot 10 - 2\cdot 0}{\epsilon} = 10$	
s^1 :	$\dfrac{\frac{-12}{\epsilon}\cdot 6 - \epsilon\cdot 10}{\frac{-12}{\epsilon}} = 6$		
s^0 :	$\dfrac{6\cdot 10 - \frac{-12}{\epsilon}\cdot 0}{6} = 10$		

sign changes in the first column ⇒ unstable.

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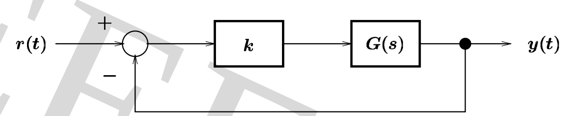
- Example 1. Consider



where k is a positive constant and

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad \zeta > 0,\ \omega_n > 0$$

- What of k can destabilize the system?

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Examples

- TF and characteristic equation.

$$\frac{Y(s)}{R(S)} = \frac{kG(s)}{1 + kG(s)}$$

$$s^2 + 2\zeta\omega_n s + (k + 1)\omega_n^2 = 0$$

- R-H table.

$$\begin{array}{lll} s^2: & 1 & (k+1)\omega_n^2 \\ s^1: & 2\zeta\omega_n & \\ s^0: & (k+1)\omega_n^2 & \end{array}$$

$\Rightarrow$ System is stable for any $k > 0$.

Examples

- Example 2. Consider with $G(s)$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{a}{s + a}, \quad k, \zeta, \omega_n, a > 0$$

- Characteristic equation.

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(s + a) + k\omega_n^2 a = 0$$

Collecting terms.

$$s^3 + (2\zeta\omega_n + a)s^2 + (\omega_n^2 + 2\zeta\omega_n a)s + (k+1)\omega_n^2 a = 0$$

Examples

- R-H table.

$$\begin{array}{lll} s^3: & 1 & \omega_n^2 + 2\zeta\omega_n a \\ s^2: & 2\zeta\omega_n + a & (k+1)\omega_n^2 a \\ s^1: & b_1 & \\ s^0: & (k+1)\omega_n^2 a & \end{array}$$

where

$$b_1 = \omega_n^2 + 2\zeta\omega_n - \frac{(k + 1)\omega_n^2 a}{2\zeta\omega_n + a}$$

- Therefore, the system is stable for $k > 0$ such that $b_1 > 0$.

Summary and Remarks

- Routh-Hurwitz stability test based on Hurwitz determinants.

  Routh simplified the into a tabular method.

- R-H test is useful for quick answers if the transfer function or characteristic equation is exactly known.

- Root locus provides a better method for visualizing the behavior of the roots of the .

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