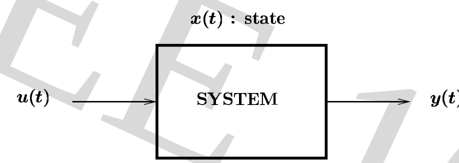
Next Topics

- Stability.
  Types of . BIBO stable vs. BIBS stable.
- Routh- test.
- Root locus and the of roots with forward gain.
- For today,
  we will look at the different aspects of stability.

Types of Stability

- BIBO (bounded input-bounded output) .

  If the input $u(t)$ to the system is bounded, then the output of the system $y(t)$ would also be .

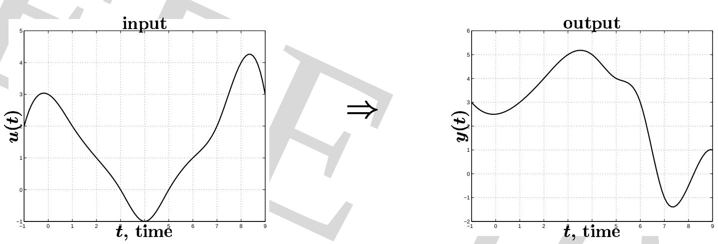


$$u(t) \text{ bounded} \quad \Rightarrow \quad y(t)$$

$$|u(t)| \leq N < \infty, \quad |y(t)| \leq M < \infty,$$
$$t \geq t_0 \qquad\qquad t \geq t_0$$

Types of Stability

- We are not concerned with what inside the system (i.e., what is happening with the state variables).



- BIBO stable = stable.

Types of Stability

- BIBS (bounded input-bounded state) stability.

  If the input $u(t)$ to the system is bounded, then the norm of the system state $x(t)$ also be bounded.

$$u(t) \text{ bounded} \quad \Rightarrow \quad x(t) \text{ bounded}$$

$$|u(t)| \leq N < \infty, \quad \|x(t)\| \leq M < \infty,$$
$$t \geq t_0 \qquad\qquad t \geq t_0$$

$\|x(t)\|$ vector norm of $x(t)$.

- BIBS stable = stable.

Types of Stability

- **Why two types of stability?**

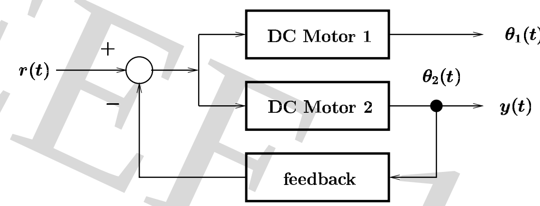
  Actually, there are other types of stability.
  BIBO and BIBS are only ones useful to us at the moment.

- **What is the importance of stability?**

- **Which one implies the other?**
  - does BIBO stability imply BIBS stability?
  - does BIBS stability BIBO stability?

BIBO vs. BIBS Stability

- **Example 1.** output.

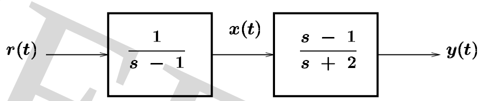


- **The feedback system only Motor 2.**

  $r(t)$ to $y(t)$ is BIBO stable. However,
  $r(t)$ to $x(t) = [\theta_1(t)\ \theta_2(t)]^T$ is not BIBS stable.

BIBO vs. BIBS Stability

- **Example 2. Pole-zero cancellation.**



- **Input to output vs. .**

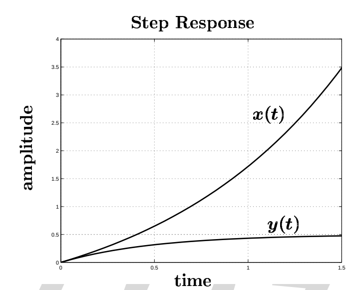
$$\frac{Y(s)}{R(s)} = \frac{1}{s-1}\cdot\frac{s-1}{s+2} = \frac{1}{s+2} \Rightarrow \text{BIBO}$$

$$\frac{X(s)}{R(s)} = \frac{1}{s-1} \Rightarrow \text{not BIBS}$$

BIBO vs. BIBS Stability

- **Matlab exercise.**

```
>> n1 = 1; d1 = [1 -1];
>> n2 = [1 -1]; d2 = [1 2];
>> [n,d] = series(n1,d1,n2,d2);
>> step(n1,d1)
>> hold
Current plot held
>> step(n,d)
```



- **State is unstable even the output is stable.**

Stability Tests

- How do we the stability of a system?

  Try out all possible bounded inputs, and then see if the system is BIBO or BIBS stable?

  There must be a better way.

- Time domain techniquies.
  - Routh-Hurwitz test.
    indicates the number of poles in the RHP.
  - root locus.
    plot of the variation of the location of the poles with the gain.

Stability Tests

- domain related techniques.
  - Nyquist criterion.
    indicates the difference between the number of poles and the number of zeros in the RHP.
  - Bode plots.
    stability check for phase systems.

- Other techniques. Lyapunov's criterion.
  - based on the energy function of a system.
  - the energy function is a system state.
  - if goes to zero, then state goes to zero.

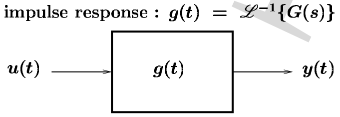
Stability of LTI Systems

- Theorem. Suppose a transfer function common factors removed.

  The system is BIBO stable if and only if the transfer function has poles inside the LHP (excluding the imaginary axis).

- Proof. $\Rightarrow$

impulse response : $g(t) = \mathscr{L}^{-1}\{G(s)\}$



Stability of LTI Systems

- Using time domain .

$$y(t) = \int_0^\infty u(t-\tau)g(\tau)d\tau$$
$$|y(t)| \le \int_0^\infty |u(t-\tau)||g(\tau)|d\tau \le N\int_0^\infty |g(\tau)|d\tau$$

BIBO stable $\Rightarrow \int_0^\infty |g(\tau)|d\tau < \infty$

- Now, we will try to get a . We will assume that poles of system are in the RHP.

Stability of LTI Systems

- However,

$$G(s) = \int_0^\infty g(t)e^{-st}dt, \; s = \sigma + j\omega$$

$$|G(s)| \leq \int_0^\infty |g(t)||e^{-st}|dt$$

If $G(s)$ has poles on the RHP (including the imaginary axis, $\sigma \geq 0$),

$$\infty \leq \int_0^\infty |g(t)||e^{-\sigma t}|dt \leq \int_0^\infty |g(t)|dt$$

$\square$ Contradiction.

Thus, BIBO stability implies that the are in the LHP.

Stability of LTI Systems

- Proof. $\Leftarrow$

Assume that system poles are in the LHP.
– perform expansion.
– use inverse Laplace transform.

Thus, if the system poles are all in the LHP, the system is BIBO stable.

- This is useful if we want to system.

Just place all the system poles in the LHP, and the system would be BIBO stable.

Summary

- Two types of stability.
BIBO stability and BIBS stability.

- Stability of systems.
Relationship of poles and system stability.

- Stability tests.
– -Hurwitz test.
– root locus.