

## What Do We Have for Today?

- Continue performance specifications.
- Time domain exercises for systems.
- Characteristic equation.
- Dominant poles and issues.

## Time Domain Exercises

- Consider

$$G(s) = \frac{1}{s^2 + 0.6s + 1}$$

- Q : What is  $\omega_n$ ,  $\zeta$ ,  $\omega_d$  and  $\sigma$ ? How do these affect the response?
- Matlab and roots of a polynomial.  
Example. Determine the of  $s^2 + 0.6s + 1$ .

```
>> roots([1 0.6 1])
```

## Time Domain Exercises

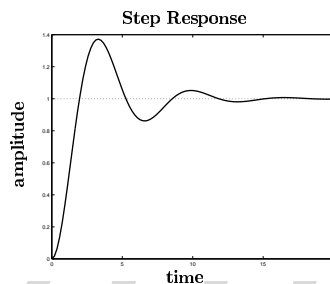
- Vary the damping factor  $\zeta$ . Step response of  $G(s)$ .

```
>> roots([1 0.6 1])
```

```
ans =
```

```
-0.3000 + 0.9539i  
-0.3000 - 0.9539i
```

```
>> step(1, [1 0.6 1])
```



## Time Domain Exercises

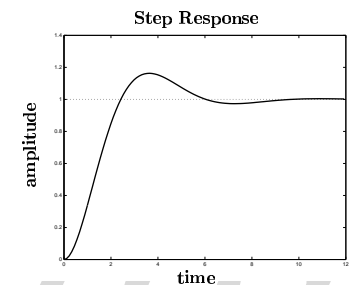
- Increase the .

```
>> roots([1 1 1])
```

```
ans =
```

```
-0.5000 + 0.8660i  
-0.5000 - 0.8660i
```

```
>> step(1, [1 1 1])
```

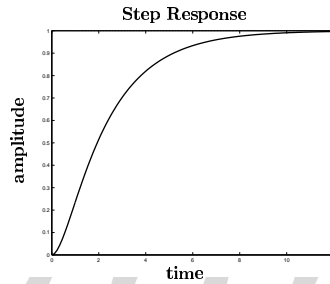


## Time Domain Exercises

```

>> roots([1 2.5 1])
ans =
-2.0000
-0.5000

>> step(1, [1 2.5 1])
    
```

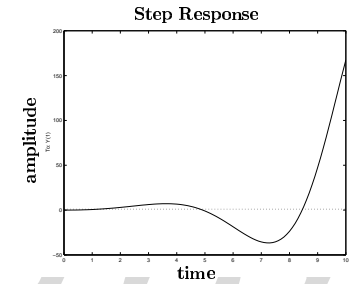


## Time Domain Exercises

```

>> roots([1 -1 1])
ans =
0.5000 + 0.8660i
0.5000 - 0.8660i

>> step(1, [1 -1 1], 0:0.1:10)
    
```

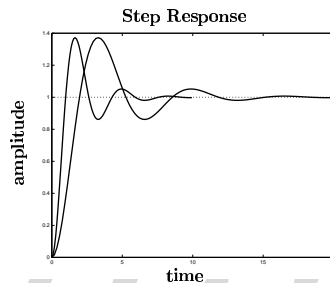


## Time Domain Exercises

- Increasing the frequency  $\omega_n$ .

```

>> wn = 1; z = 0.3;
>> step(wn^2, [1 2*z*wn wn^2])
>> hold
Current plot held
>> wn = 2;
>> step(wn^2, [1 2*z*wn wn^2])
    
```

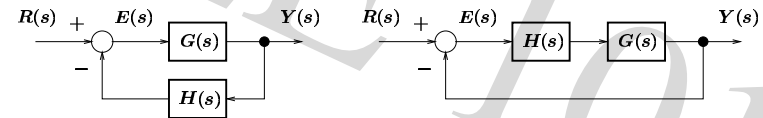


## Characteristic Equation

- Error equation

$$E(s) = \frac{1}{1 + H(s)G(s)} R(s)$$

- We have the same error for these two systems.



- Error equation and

$$\text{function : } E = SR.$$

## Characteristic Equation

- Poles of the transfer function from  $R(s)$  to  $E(s)$ .  
 $\Rightarrow$  roots or zeros of  $1 + H(s)G(s)$ .

- Characteristic equation:  $1 + H(s)G(s) = 0$ .

- Example.

$$G(s) = \frac{1}{s^2 + 0.6s + 1}, \quad H(s) = 1$$

## Characteristic Equation

- Error equation and characteristic equation.

$$\frac{E(s)}{R(s)} = \frac{s^2 + 0.6s + 1}{s^2 + 0.6s + 2}$$

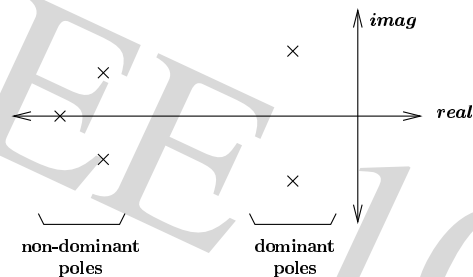
$$1 + H(s)G(s) = 0 \Rightarrow s^2 + 0.6s + 2 = 0$$

roots without feedback :  $s = -0.3 \pm j0.9539$   
 roots with feedback :  $s = -0.3 \pm j1.3820$

- Therefore, feedback
  - modifies system behavior.
  - moves the location of the poles.

## Dominant Poles

- Location of poles on the  $s$ -plane.



- dominant poles : decay slowly.
- non-dominant poles : decay rapidly.

## Dominant Poles

- In some cases, we need to worry only about dominant poles and ignore non-dominant poles.

- Example. Compare the response of

$$G_1(s) = \frac{1}{s^2 + 0.6s + 1}$$

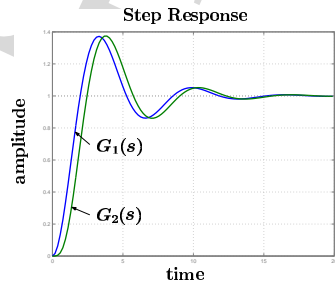
and

$$G_2(s) = G_1(s) \frac{9}{s^2 + 4.2s + 9}$$

## Dominant Poles

- Step response

```
>> n1 = 1; d1 = [1 0.6 1]; step(n1, d1)
>> [n2, d2] = series(n1, d1, 9, [1 4.2 9]);
>> step(n2, d2)
```

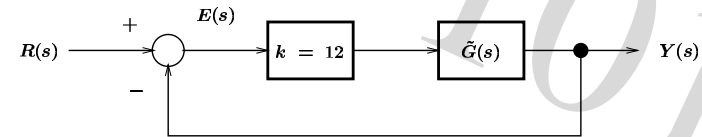


## Dominant Poles

- What happens if you ignore 'insignificant poles?'

Suppose the actual plant model is  $G(s) = G_2(s)$ . However, in designing the control, non-dominant poles were ignored, i.e., we use  $\tilde{G}(s) = G_1(s)$ .

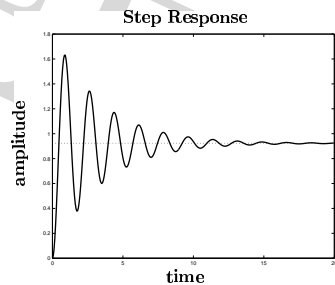
- Design the



## Dominant Poles

- Step response of the design with the plant  $\tilde{G}(s)$ .

```
>> [nc1, dc1] = cloop(12*n1, d1, -1);
>> step(nc1, dc1)
```

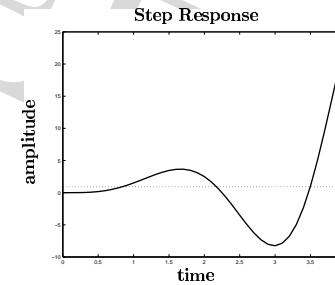


⇒ We have a system.

## Dominant Poles

- Check using the actual plant model,  $G(s)$ .

```
>> [nc2, dc2] = cloop(12*n2, d2, -1);
>> step(nc2, dc2, [0:0.1:4])
```



⇒ The is unstable.

## Dominant Poles

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- The step responses of low-order systems and high-order systems may be similar.

- Good thing. This can simplify the design of the step response of a high-order system.

Inspect the  $T_r$ ,  $T_d$ , natural frequency, etc.

There is 'little' difference between the step responses of the second-order system  $G_1(s)$  and the fourth-order system  $G_2(s)$ .

## Dominant Poles

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- Not so good thing. This can lead to identifying the system order, and the system model.

Inspect the  $T_r$ ,  $T_d$ , natural frequency, etc.

In designing a control system for  $G_2(s)$ , one may incorrectly model the system as  $G_1(s)$ , and thus lead to a poorly designed controller.

- An accurate model is important in control systems design.

## Summary

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- Varying  $\zeta$  and  $\omega_n$  varies the response of a second-order system.

- Characteristic equation, and how it affects a system.

- Dominant poles. Making the right choice.