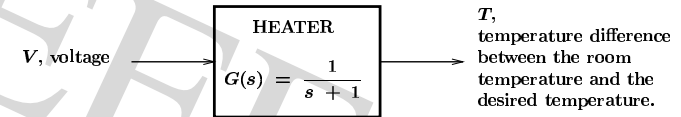## Where are we? Where are we heading?
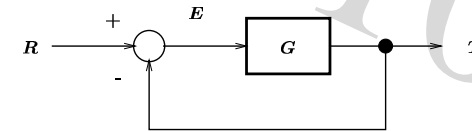
- What have we done so far?
  - we are now _____ with control systems.
  - we know what a model is and how to get it.
  - we have the _____ to deal with the math.

- What will be looking at next?
  - we will start investigating systems in detail.
  - we will work first with steady-state.
  - time domain and _____ domain characterizations.

- For today, look at system type and steady-state error.

## System Type

- Simple _____ model (first-order, type 0).



$V$, voltage → HEATER, $G(s) = \dfrac{1}{s+1}$ → $T$, temperature difference between the room temperature and the desired temperature.

- Add _____ .



$R \xrightarrow{+} \bigcirc \xrightarrow{E} \boxed{G} \bullet \rightarrow T$

## System Type

- Steady-____ error.

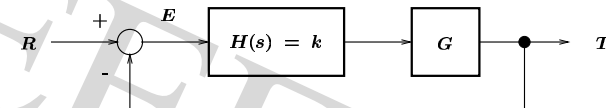$$\frac{E}{R} = \frac{R - T}{R} = \frac{1}{1 + G} = \frac{s + 1}{s + 2}$$

- For a step input : $r(t) = u(t)$, then

$$e_{ss} = \lim_{s \to 0} s \frac{s + 1}{s + 2} \frac{1}{s} = \frac{1}{2}$$

- For a type 0 system.
  $\Rightarrow$ non-zero _____ error in response to a step input

## System Type

- Insert additional gain.



$R \xrightarrow{+} \bigcirc \xrightarrow{E} \boxed{H(s) = k} \rightarrow \boxed{G} \bullet \rightarrow T$

```
>> [n,d] = cloop(5, [1 1], -1)   % gain k = 5
>> step(n, d)
```

- Steady-state error.

$$\frac{E}{R} = \frac{s + 1}{s + 1 + k}$$

- For a step : $r(t) = u(t)$, then

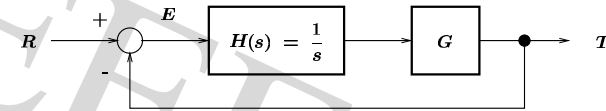$$e_{ss} = \lim_{s \to 0} s \, \frac{s + 1}{s + 1 + k} \, \frac{1}{s} = \frac{1}{1 + k}$$

- Observations.
  - gain the steady-state error.
  - infinite gain is required for 'perfect' tracking.

---

- Try something else $\to$ add an integrator.



$$\frac{N(s)}{D(s)} = \frac{1}{s} \frac{1}{s + 1} = \frac{1}{s^2 + s}$$

```
>> [n,d] = cloop(1, [1 1 0], -1)
>> step(n, d)
```

---

- Steady-state .

$$\frac{E}{R} = \frac{1}{1 + HG} = \frac{s^2 + s}{s^2 + s + 1}$$

- For a step input : $r(t) = u(t)$, then

$$e_{ss} = \lim_{s \to 0} s \, \frac{s^2 + s}{s^2 + s + 1} \, \frac{1}{s} = 0$$

- System type is now .
  The system can track a step input with $e_{ss} = 0$.

---

- System 0 :

$$\lim_{s \to 0} GH = K_p < \infty$$

$$r(t) = u(t)$$
$$\Rightarrow e_{ss} = \lim_{s \to 0} s \, \frac{1}{1 + GH} \, \frac{1}{s} = \frac{1}{1 + K_p}$$

$$r(t) = tu(t)$$
$$\Rightarrow e_{ss} = \lim_{s \to 0} s \, \frac{1}{1 + GH} \, \frac{1}{s^2} = \infty$$

- type 1.

$$\lim_{s\to 0} sGH = K_v < \infty \text{ and } \lim_{s\to 0} GH = K_p = \infty$$

$$r(t) = u(t)$$
$$\Rightarrow e_{ss} = 0$$

$$r(t) = tu(t)$$
$$\Rightarrow e_{ss} = \lim_{s\to 0} s \frac{1}{1+GH}\frac{1}{s^2} = \frac{1}{K_v}$$
$$r(t) = \frac{t^2}{2}u(t)$$
$$\Rightarrow e_{ss} = \lim_{s\to 0} s \frac{1}{1+GH}\frac{1}{s^3} = \infty$$

---

- A        at system type.



- Transfer function.

$$G(s) = k\frac{(s-z_1)\dots(s-z_m)}{s^j(s-p_1)\dots(s-p_n)}$$
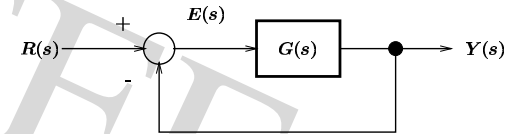
where $j$ is the system    .

---

- Error equation.

$$E(s) = \frac{1}{1+G(s)}R(s)$$

$$E(s) = \frac{1}{1 + k\dfrac{(s-z_1)\dots(s-z_m)}{s^j(s-p_1)\dots(s-p_n)}}R(s)$$

$$E(s) = \frac{s^j(s-p_1)\dots(s-p_n)}{\left\{\begin{array}{l} s^j(s-p_1)\dots(s-p_n) + \\ k(s-z_1)\dots(s-z_m)\end{array}\right\}}R(s)$$

---

- For        ,

$$E(s) = \frac{s^j\displaystyle\prod_{q=1}^{n}(s-p_q)}{s^j\displaystyle\prod_{q=1}^{n}(s-p_q) + k\displaystyle\prod_{r=1}^{m}(s-z_r)}R(s)$$

- Steady-state error.

$$e_{ss} = \lim_{s\to 0} sE(s)$$

- Step input, $r(t) = u(t)$.

$$e_{ss} = \lim_{s \to 0} s \; \frac{s^j \displaystyle\prod_{q=1}^{n}(s - p_q)}{s^j \displaystyle\prod_{q=1}^{n}(s - p_q) + k \displaystyle\prod_{r=1}^{m}(s - z_r)} \; \frac{1}{s}$$

- Error        for a step input.
  - system type 0 : $e_{ss} =$ finite constant.
  - system type 1 or       : $e_{ss} = 0$.

- Ramp input, $r(t) = tu(t)$.

$$e_{ss} = \lim_{s \to 0} s \; \frac{s^j \displaystyle\prod_{q=1}^{n}(s - p_q)}{s^j \displaystyle\prod_{q=1}^{n}(s - p_q) + k \displaystyle\prod_{r=1}^{m}(s - z_r)} \; \frac{1}{s^2}$$

- Error constants for a       input.
  - system type 0 : $e_{ss} = \infty$.
  - system type 1 : $e_{ss} =$ finite constant.
  - system type 2 or       : $e_{ss} = 0$.

- Same procedure for       input.
  - system type 0 : $e_{ss} = \infty$.
  - system type 1 : $e_{ss} = \infty$.
  - system type 2 : $e_{ss} =$ finite constant.
  - system   type   or greater : $e_{ss} = 0$.

- Observations.
  - system type affects steady-state error.
  -       function affects steady-state error.

- One can determine steady-state error constant immediately by simply looking at the system type and input function.

- One will know what system type is needed drive the steady-state error to       for a known input.

- Add            to increase the system type and consequently make the steady-state error zero.