Today's EEE 101 Lecture

- Comparison of block diagrams and SFGs.
- transformations.
- Mason gain rule.
- Summary.

Signal Flow Graphs

- relationship depicted by line segments between input and output nodes.

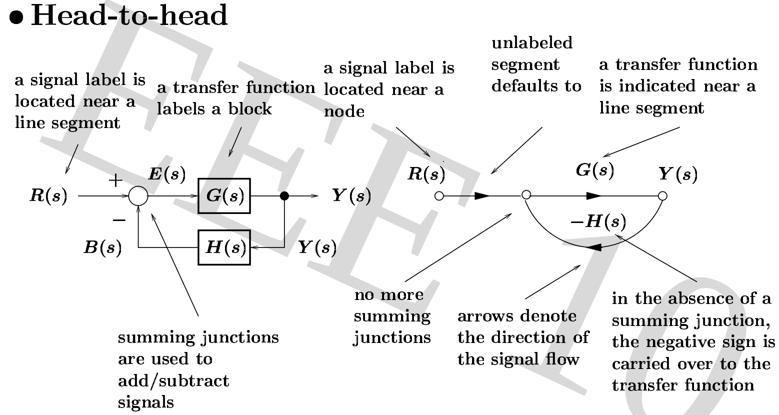
- Signal flow is unidirectional from .
- Simpler representation of a system compared to block diagrams.

Signal Flow Graphs

- Complex systems are represented by multiple and interconnected graphs.
- may be reduced by SFG transformations.
- Mason can also be applied to simplify SFGs.

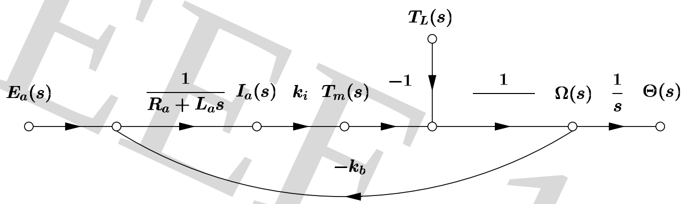
Signal Flow Graphs

- Head-to-head



Signal Flow Graphs

- **Example. SFG of the system.**



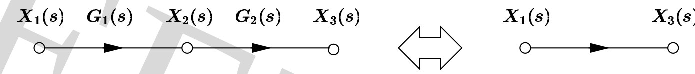
where

$\Omega(s) = \mathscr{L}[\dot{\theta}(t)]$

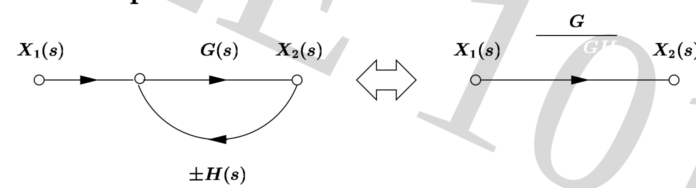
$\Theta(s) = \mathscr{L}[\quad]$

SFG Transformations

- **Series blocks**



- **Feedback loop**



Mason Gain Rule

- **Mason gain rule**

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

where

y_{in} = input node variable

y_{out} = output node variable

M = gain between y_{in} and y_{out}

N = number of

M_k = gain of k-th forward path

Δ = determinant of the whole SFG

Δ_k = Δ for the SFG which is
with the k-th forward path.

Mason Gain Rule

- **Computation of the determinant Δ.**

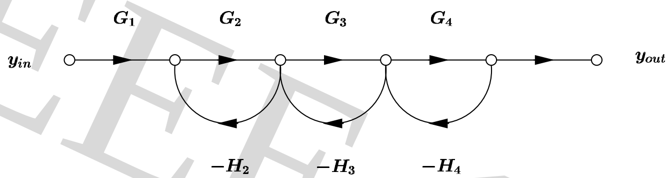
$$\Delta = 1 - \sum_m P_m^1 + \sum_m P_m^2 - \sum_m P_m^3 + \ldots$$

where P_m^r is the gain product of the m-th combination of r non-touching loops.

- **Two parts of the SFG are if they do not share a common node.**

Mason Gain Rule

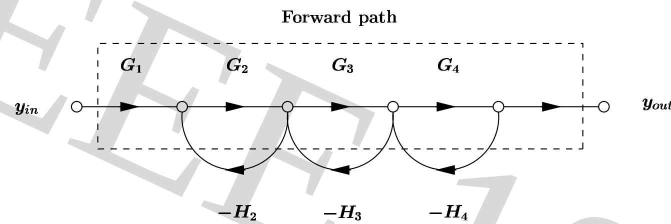
- Example. SFG reduction using .



- The SFG has
  - one path.
  - three loops.
  - one pair of non-touching loops.

Mason Gain Rule

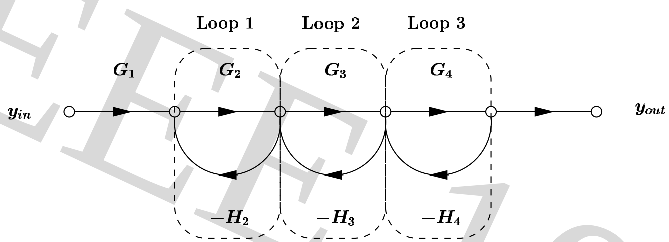
- Forward path.



$$M_1 =$$
$$\Delta_1 = 1$$

Mason Gain Rule

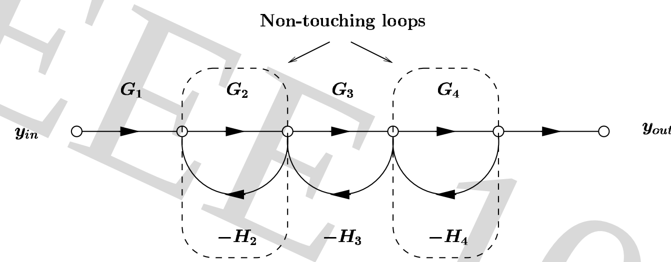
- Individual loops.



$$\text{Loop 1}: P_1^1 = -G_2H_2$$
$$\text{Loop 2}: P_2^1 =$$
$$\text{Loop 3}: P_3^1 = -G_4H_4$$

Mason Gain Rule

- Pairs of .



$$P_1^2 = (-G_2H_2)(-G_4H_4)$$
$$=$$

Mason Gain Rule

- **Determinant of the SFG.**

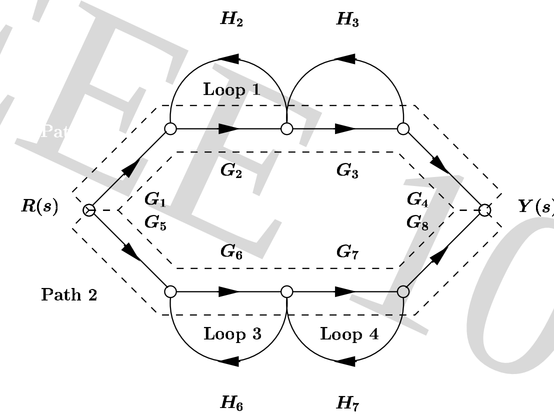
$$\Delta = 1 - (P_1^1 + P_2^1 + P_3^1) +$$

- **Transfer function**

$$\frac{y_{out}}{y_{in}} = \frac{M_1\Delta_1}{\Delta}$$
$$= \frac{G_1G_2G_3G_4}{1 + G_2H_2 + G_3H_3 + \quad + G_2H_2G_4H_4}$$

Mason Gain Rule

- **Example. Interacting system.**



Mason Gain Rule

- **Two forward paths from input to output.**

$$M_1 = G_1G_2G_3G_4$$
$$M_2 =$$

- **Individual loops.**

$$P_1^1 = G_2H_2, \quad P_2^1 = G_3H_3,$$
$$P_3^1 = G_6H_6, \quad P_4^1 = G_7H_7$$

Mason Gain Rule

- **of non-touching loops.**

For conciseness, let

$$\text{Loop 1}: L_1 = G_2H_2, \quad \text{Loop 2}: L_2 = G_3H_3,$$
$$\text{Loop 3}: L_3 = G_6H_6, \quad \text{Loop 4}: L_4 = G_7H_7$$

Then,

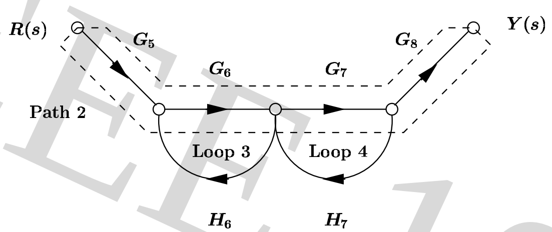
$$P_1^2 = L_1L_3, \quad P_2^2 = L_1L_4,$$
$$P_3^2 = L_2L_3, \quad P_4^2 = L_2L_4$$

- **of the SFG.**

$$\Delta = 1 - (P_1^1 + P_2^1 + P_3^1 + P_4^1) + (P_1^2 + \quad + P_4^2)$$

Mason Gain Rule

- With the upper forward path removed,

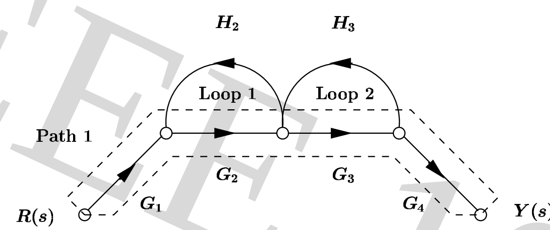


only remain. Thus,

$$\Delta_1 = 1 - (L_3 + L_4)$$

Mason Gain Rule

- With the lower forward path removed,



only Loop remain. Thus,

$$\Delta_2 = 1 - (L_1 + L_2)$$

Mason Gain Rule

- Transfer function

$$\frac{Y(s)}{R(s)} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

$$= \frac{\left\{\begin{array}{l} G_1G_2G_3G_4(1 - L_3 - L_4) + \\ G_5G_6G_7G_8(1 - L_1 - L_2) \end{array}\right\}}{\left\{\begin{array}{l} 1 - L_1 - L_2 - L_3 - L_4 + \\ L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4 \end{array}\right\}}$$

- Note : Mason gain rule care.

Summary

- What are SFGs?
  How are they different from ?
- Mason gain rule; an important reduction technique.
- How do Mason gain rule?