Upcoming Lecture Features

- Main things we will be looking at.
  - block diagrams
  - signal flow graphs
  - Mason gain rule
  - general control system

- For today's lecture, we have
  - diagrams
  - interpreting a function

What is a Transfer Function?

- Dynamic systems are represented by equations.

- Laplace transformation simplifies solutions to differential equations.

- In control systems, variables are to achieve a specific or desired behavior.

What is a Transfer Function?

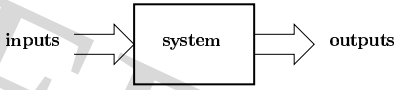
- Relationship between controlling variable(s) and controlled variable(s) are needed.

- usually represented by a transfer function.

- Controlling variable - input
  Controlled variable - state, output
  Transfer function - of input to output of a dynamic system.

Block Diagrams

- Input-output usually depicted by blocks.



- Signal flow is from input to output.

- Simple systems can be represented by a single block.

Block Diagrams

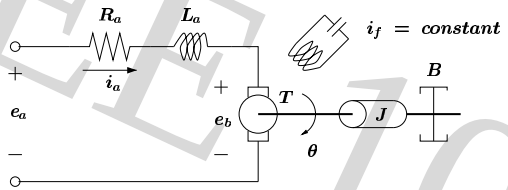
- Complex systems are represented by multiple and interconnected blocks.

  Room [illegible] example.
  Hard to model the system as one complex system.
  Break it down into [illegible] subsystems.

- Block diagrams may be reduced by block diagram transformations.
  - [illegible] connected
  - single feedback
  - parallel connected

Block Diagrams

- Example. DC [illegible].
  - small rotor inertia
  - high torque-to-inertia ratio



- Focus on armature-current [illegible].

Block Diagrams

- Electrical equations.

$$e_a = R_a i_a + L_a \dot{i}_a + e_b \quad \text{(KVL)}$$
$$e_b = k_b \dot{\theta} \quad \text{(back-EMF)}$$

- [illegible] equations.

$$T = k_i i_a \quad \text{(motor torque)}$$
$$T = J\ddot{\theta} + B\dot{\theta} \quad \text{(sum torques)}$$

- Variables.

  input (e_a) $\Rightarrow$ state ($\dot{\theta}$ and i_a) $\Rightarrow$ output (θ)

Block Diagrams

- From [illegible] equation,

$$e_a = R_a i_a + L_a \dot{i}_a + e_b$$
$$\Rightarrow R_a i_a + L_a \dot{i}_a = e_a - e_b$$
$$\mathscr{L} \Rightarrow R_a I_a + L_a s I_a = E_a - E_b \quad \text{(ignore I.C.)}$$
$$\Rightarrow I_a = \frac{1}{R_a + sL_a}(E_a - E_b)$$

- From back-EMF [illegible],

$$e_b = k_b \dot{\theta}$$
$$\mathscr{L} \Rightarrow E_b = k_b \Omega$$

Block Diagrams

- From motor equation,

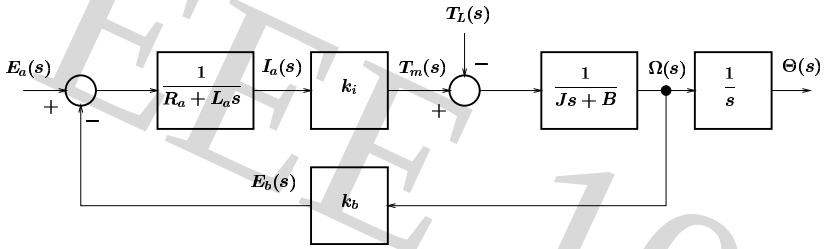
$$T_m = k_i i_a$$
$$\mathscr{L} \Rightarrow T_m = k_i I_a$$

- From of torques equation,

$$T_m = J\ddot{\theta} + B\dot{\theta} + T_L$$
$$\Rightarrow T_m - T_L = J\ddot{\theta} + B\dot{\theta} = J\dot{\omega} + B\omega$$
$$\mathscr{L} \Rightarrow T_m - T_L = Js\Omega + B\Omega$$
$$\Rightarrow \Omega = \frac{1}{sJ + B}(T_m - T_L)$$

Block Diagrams

- Block diagram of a system

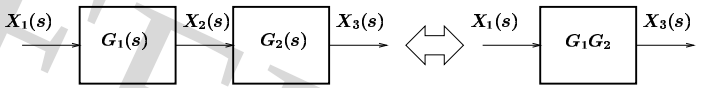


where

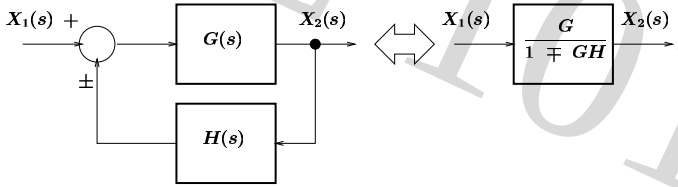
$$\Omega(s) = \mathscr{L}[\theta(\dot{t})]$$
$$\Theta(s) = \mathscr{L}[\theta(t)]$$

Block Diagram Transformations

- Series blocks

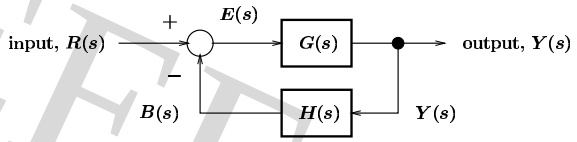


- loop



Block Diagram Transformations

- Closed-loop function



error equation : $E(s) = R(s) - B(s)$

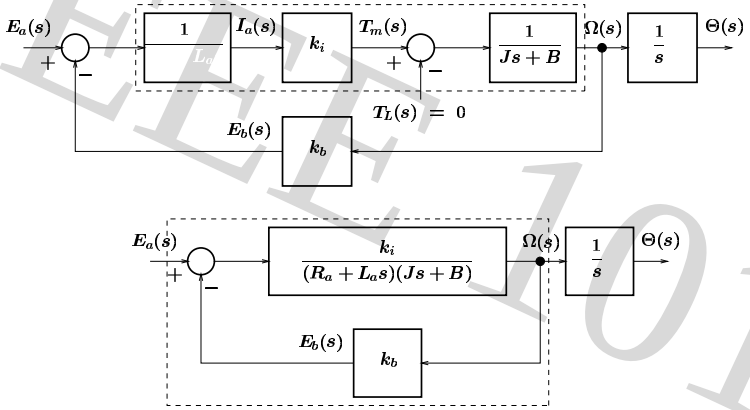
$$\Rightarrow E(s) = R(s) - H(s)Y(s)$$
$$: Y(s) = G(s)E(s)$$
$$\Rightarrow Y(s) = G(s)[R(s) - H(s)Y(s)]$$

solving for $Y(s)$: $Y(s)[1 + G(s)H(s)] = G(s)R(s)$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

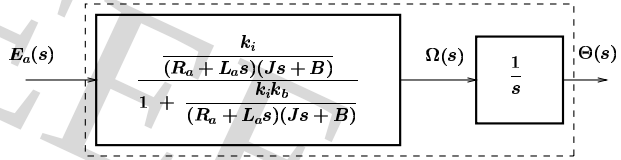
Block Diagram Transformations

- Example. DC servomotor with no load (i.e., $T_L = 0$).

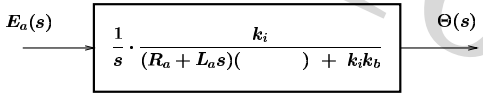


Block Diagram Transformations

- el reduction ...



- Transfer function of a DC servomotor



Summary

- There is more than one way to a system.
  The choice depends on what you want to do.

- Transfer function representation.
  One , one t representation. Very useful in control system feedback design.

- Block diagrams are also used to represent systems.
  You usually come up with first, then simplify to a transfer function.