Today's EEE 101 Lecture

- Mathematical of dynamic systems.
- What is a model? What is a dynamic system?
- Systems we will be looking at.
  - electrical systems
  - systems (translational and rotational)
  - electromechanical systems
  - thermal systems and systems

What is a Model?

- Dictionary definition. Model.
  - n. A small object, usually built to , that represents in detail another, often larger object.
  - n. A preliminary work or construction that serves as a plan from which a final product is to be made.
  - n. A schematic of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics.
  - n. A style or design of an item.
  - n. One serving as an to be imitated or compared.

What is a Model?

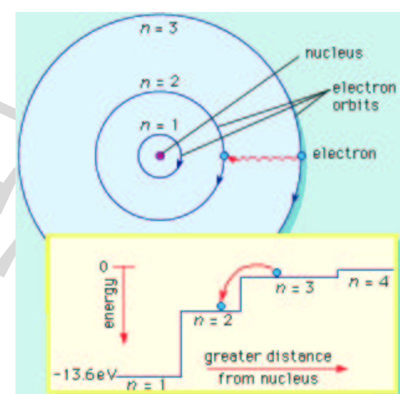
- Other definitions of a model.
  - n. A employed to display merchandise, such as clothing or cosmetics.
  - n. Zoology. An animal whose appearance is copied by a mimic.
- Modeling (computer terms).
  Generally, the process of a real-world object or phenomenon as a set of mathematical equations. More specifically, the term is often used to describe the process of representing 3-D objects in a computer.

What is a Model?

- Verbal model.

  Bohr atomic model.

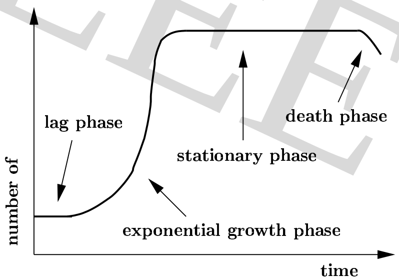
  Bohr proposed that atoms are restricted to certain fixed (quantized) orbits. An electron can jump, suddenly, between these orbits by or emitting a photon the appropriate precise .



What is a Model?

- Graphical model.

Bacteria growth model.



The growth shown is associated with simplistic conditions known as a batch culture.

Single bacterial culture introduced into and growing in a fixed volume with a fixed (limited) amount of nutrient.

What is a Model?

- Mathematical model.

Radioactive decay.
In 1903, Rutherford and Soddy proposed the of radioactive change which states that rate of decay of radioactive matter was proportional to the present. Mathematically, we say

$$\text{rate of} \quad \text{of } [A] \; = \; -\, k \cdot [A]$$

where $[A]$ and k are the concentration and the rate of decay of substance A, respectively. Thus,

$$[A](t) \; = \; [A](0)b^t \qquad 0 \; < \; b \; < \; 1$$

What is a Model?

- We will be using models in EEE 101.

  We have tools (differential equations and Laplace transforms) to deal with mathematical equations.

- May sometimes start from a model or a graphical model and an approximate mathematical model.

- Other control techniques exist for directly dealing with verbal and graphical models.

Dynamic Systems

- Systems where variables (state variables) change with respect to time.

- Dynamic systems are usually with differential equations (or possibly, difference equations).

  Example. Robotic manipulator model.

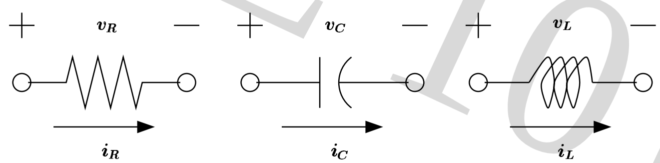
$$D(q)\ddot{q} \; + \; C(q, \dot{q}) \; + \; G(q) \; = \; \tau$$

- We need to able to handle equations.

Electrical Systems

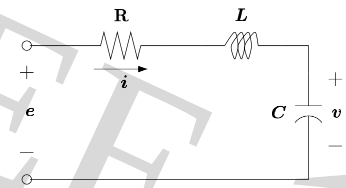
- Variables : voltage v and current i.
- Resistance, and inductance.

$$v_R = i_R R, \quad i_C = C\frac{dv_C}{dt}, \quad v_L = L\frac{di_L}{dt}$$



Electrical Systems

- Example. RLC circuit



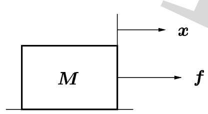
Two first-order equations

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad i = C\dot{v}$$

$$e = \quad + L\frac{di}{dt} + v \quad \Rightarrow \quad e = iR + L\dot{i} + v$$

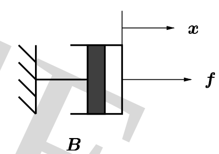
Mechanical Systems

- Translational variables :
  - − , x
  - − velocity, $\dot{x} = \frac{dx}{dt}$
  - − force, f
- Mass : $f = M\ddot{x}$

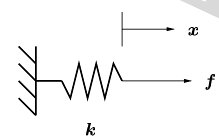


Mechanical Systems

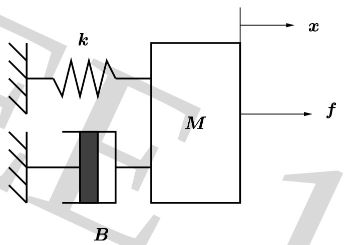
- Friction (viscuous): $f =$



- (linear): $f = kx$



- Example. Mass, and damper system

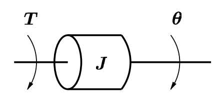


Summing forces along the horizontal

$$f = M\ddot{x} + kx + B\dot{x}$$

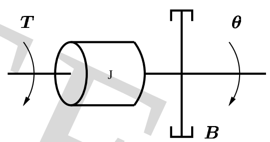
- Rotational
  Variables :
  - angle, θ
  - angular velocity, $\dot{\theta}$
  - torque, T

- Inertia (rotational) : $T = J\ddot{\theta}$

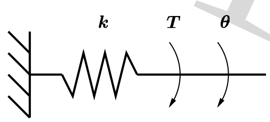


for a circular rotating about a geometric axis
$J = \frac{1}{2}Mr^2$

- on (rotational): $T = B\dot{\theta}$

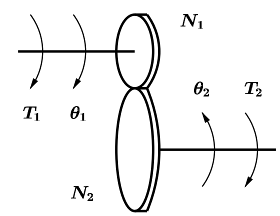


- Spring (torsional): $T =$



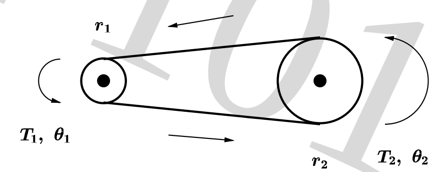
- Gears.

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\dot{\theta}_2}{\dot{\theta}_1}$$



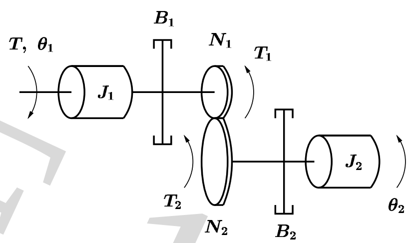
- Belt-driven .

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{r_1}{r_2}$$



Mechanical Systems

- Example. Reflected load



T : applied
T_1 : applied by gear 2
T_2 : applied to shaft 2

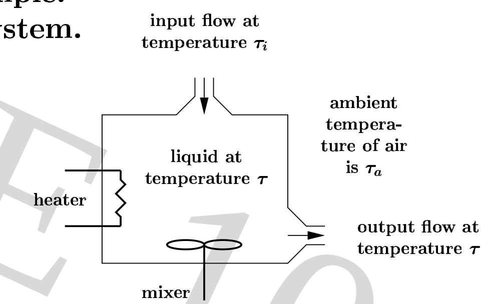
$$T_2 = J_2\ddot{\theta}_2 \ + \ B_2\dot{\theta}_2$$
$$T = J_1\ddot{\theta}_1 \ + \ B_1\dot{\theta}_1 \ + \ T_1$$

$$\Rightarrow T_1 \ = \ \frac{N_1}{N_2}T_2 \ = \ \left(\frac{N_1}{N_2}\right)^2 J_2\ddot{\theta}_1 \ + \ \left(\frac{N_1}{N_2}\right)^2 B_2\dot{\theta}_1$$

Thermal Systems and Liquid-level Systems

- Another modeling example.
  Temperature control system.

  Liquid is at a certain rate while being replaced by liquid with temperature τ_i.



- The liquid is heated by an and agitated by a mixer such that the liquid temperature is uniform inside the tank.

Thermal Systems and Liquid-level Systems

- Define

  $q_e(t) \triangleq$ heat increase supplied by the heater.
  $q_i(t) \triangleq$ heat increase from entering liquid.
  $q_l(t) \triangleq$ heat absorbed by the liquid.
  $q_o(t) \triangleq$ heat decrease from exiting liquid.
  $q_s(t) \triangleq$ heat loss through surface.

- Conservation of energy.

$$q_i(t) \ + \ q_e(t) \ = \ q_l(t) \ + \ q_o(t) \ + \ q_s(t)$$

Thermal Systems and Liquid-level Systems

- The heat absorbed by the liquid is related to the thermal capacity C and the change by

$$q_l(t) \ = \ C\frac{d\tau(t)}{dt}$$

- Let $v(t)$ be the flow rate in (and out) of the tank. If H is the specific heat of the liquid, then

$$q_i(t) \ = \ v(t)H\tau_i(t) \text{ and } q_o(t) \ = \ v(t)H\tau(t)$$

Thermal Systems and Liquid-level Systems

- The heat loss through the tank surface in terms of the thermal resistance R of the tank surface is

$$q_s(t) = \frac{\tau(t) - \tau_a(t)}{R}$$

- Combining the above equations, we get the equation model of the system.

$$q_e(t) + v(t)H\tau_i(t) = C\frac{d\tau(t)}{dt} + v(t)H\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

Thermal Systems and Liquid-level Systems

- Assuming that the flow rate $v(t)$ is at a constant value V, we get a first-order ODE.

$$q_e(t) + VH\tau_i(t) = C\frac{d\tau(t)}{dt} + VH\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

- In view of a control system,
  - $q_e(t)$ is the input,
  - $\tau_i(t)$ and $\tau_a(t)$ are disturbances, and
  - $\tau(t)$ as the output.

Why Use a Model?

- A mathematical model can be
  - used to simulate situations,
  - subjected to states that would be dangerous in reality, and
  - used as basis for synthezing controllers.

- Note. Real processes are , and getting an exact model is usually impossible.

  However, feedback is lenient (to some degree) and controllers can be designed using simple models.

  The model must capture the essential features of the plant.

Why Use a Model?

- Some definitions.
  - model.
    An approximate description of the plant used for control system design.
  - calibration model.
    A comprehensive description of the plant. It can include behaviors not used for control design but may impact performance.
  - model .
    Difference between the nominal model and the calibration model. This error may be unknown but bounds may be available.

Why Use a Model?

- The nominal model may be a simplified version of the calibration model.
- The nominal model is used in doing the .

  The value of the plant input is directly based on this model.
- The calibration model is used to verify if the control works.

  This model may be used in numerical to see if the controller performance is satisfactory.

Why Use a Model?

- Example. Robotic manipulator model.

$$D(q)\ddot{q} \;+\; C(q,\dot{q}) \;+\; G(q) \;=\; \tau$$

  - $D(q)\ddot{q}$ inertia term.
  - $C(q,\dot{q})$ coriolis and centrifugal terms.
  - $G(q)$ gravity term.
- For control purposes, $C(q,\dot{q})$ term may be neglected.

  In some instances, only the term is used in generating the control.
- When doing simulations, the model is used.

Summary

- What is a system?

  What are models? What is a mathematical model?
- Familiarize with
  - electrical systems
  - mechanical systems (translational and rotational)
- Review
  - electromechanical systems
  - thermal and liquid-level systems