Today's EE 233 Lecture

- Overview of what you should know?
- Digital control system.
- The control problem.
- Some examples on modelling.
- Summary.

Introduction

- Analysis and design of closed-loop physical systems.

  Should have learned something about this in a previous control systems course.

- Digital control is used to modify system behavior such that the desired response is obtained.

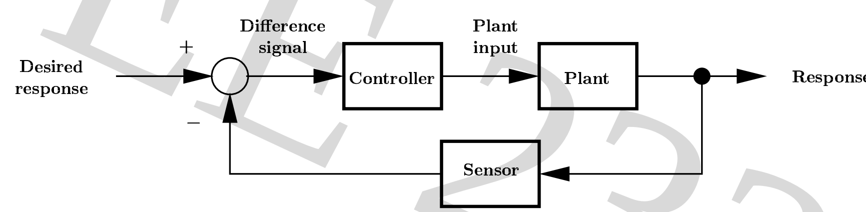
  Digital computers or controllers are inserted in the system to modify the dynamics.

Introduction

- Advantages of digital control (vs. analog control).
  - easier to modify controller dynamics.
  - no controller degradation or drift due to aging.
  - can implement exotic control schemes?
- Disadvantages of digital control (vs. analog control).
  - more expensive?
  - continuous systems are easier to grasp?
  - mathematics is a bit more involved?

Introduction

- Closed-loop system is a system wherein the inputs (forcing functions) are determined somewhat by the outputs (response) of the system.



- Input is a function of (although not entirely) the output.

Introduction

- Simple closed-loop system.
  - plant.
  - control actuator.
  - sensor(s).
  - controller (compensator or filter).

- One of the tasks of a control system designer is the design of the controller.

  The designer may also be tasked to select what sensors, actuators and controller is to be used.
  The plant is usually known (just model it).

Example of a Closed-loop System

- Pilot landing an aircraft.
  The aircraft is the plant.

- Inputs are position of aircraft control surfaces (e.g. rudder, elevators, ailerons) and engine thrust.

- The pilot functions as the sensor (perceptions of environment and instruments).

  The controller is the pilot's action based on the perceived error in the flight of the aircraft.

Example of a Closed-loop System

- For the aircraft system, the pilot is a major part of the control system (sensor, controller and actuator).

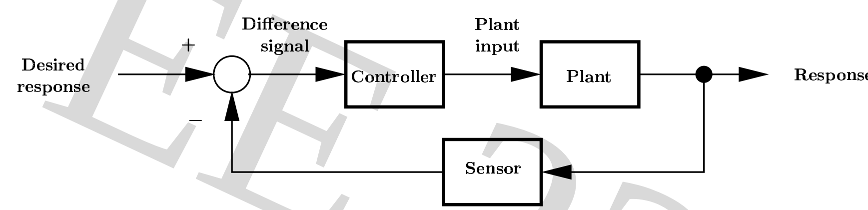
- The design of the aircraft control system corresponds to the training of the pilot based on a specific aircraft.

  Different aircrafts have different dynamics and inputs.

  Flying a fighter jet is not the same as flying a commercial passenger airliner.

Digital Control

- Consider the following control system.



- Appropriate sensors for measuring system response.

  Digital computer acting as the controller to achieve the desired response.

Digital Control

- Digital controller can be microprocessors, microcontrollers, workstations and other digital devices.
- The plant exhibits some dynamics. The dynamics will be 'programmed' into the digital controller.
- Plant dynamics are oftentimes fixed. However, we can choose the design (in some sense, programming) of the controller.

Digital Control

- We need to design the controller such that we get the desired (or satisfactory) closed-loop system response.
- In the aircraft example, let us say we want to design an automatic landing system.

  Satisfactory response can be
  - a good landing approach,
  - a comfortable ride and
  - no undue mechanical stress on the aircraft.

Digital Control

- Linear system - system where the principles of superposition and homogeneity apply.

  All physical systems are by nature nonlinear. However, most systems can be designed to operate within a linear region.
- Time-invariant system - system parameters are constant with respect to time.

  What systems are time-varying? Block of wood? Space vehicle?

Digital Control

- Continuous-time system - system where signals change continuously.
- Discrete-time system - signals change values only at discrete time instants.
- We will be looking at mostly linear time-invariant dicrete-time systems.

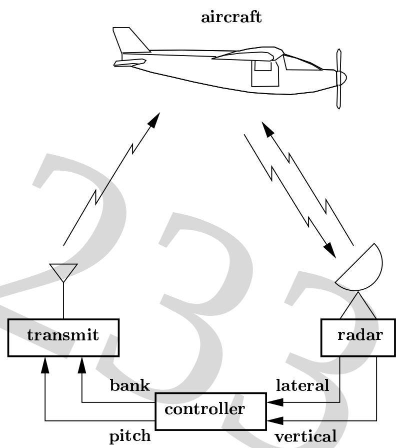
  We will achieve the control of such systems using digital control techniques.

Digital Control Basics

- Example. Automatic aircraft landing system.

- Control pitch and bank of aircraft via transmitter.

  Pitch and bank commands result in a change in aircraft position.



Digital Control Basics

- The system has three basic parts.
  - aircraft - receives commands and takes action.
  - radar unit - measures approximate aircraft position.
  - controller - determines appropriate aircraft commands.

- The controller is a digital computer.
  - lateral control - lateral position of the aircraft.
  - vertical control - altitude of the aircraft.

Digital Control Basics

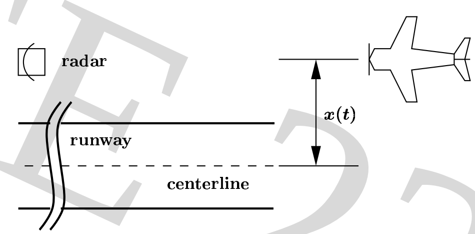
- Assume that the control is independent (decoupled).
  - bank command input - affects only the lateral position.
  - pitch command input - affects only the altitude.

- Look at the lateral position control.

- The lateral aircraft position $x(t)$ is the perpendicular distance of the aircraft from the centerline of the runway.

Digital Control Basics

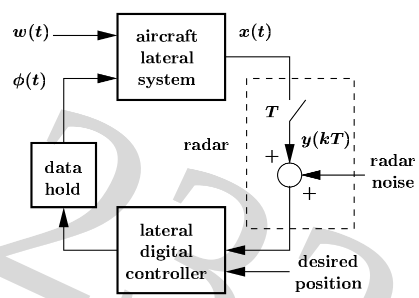
- The control system tries to force $x(t)$ to zero.



- The radar unit measures $x(t)$ every 50 ms. We have the sampled value $x(kT)$ of $x(t)$ with $T = 50\ ms$ and $k = 0, 1, 2, \ldots$.

Digital Control Basics

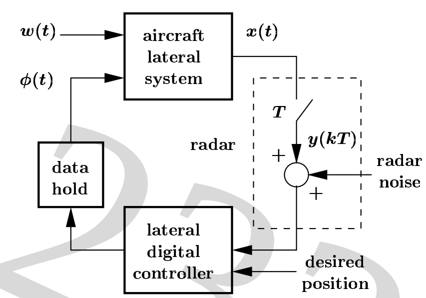
- The controller processes the sampled values and generates (also transmits) discrete-time bank commands $\phi(kT)$.



- The commands are received by the aircraft and clamped by a data hold.

Digital Control Basics

- Consequently, the bank command $\phi(t)$ remains constant at the last value until a new value is received.



- The aircraft responds to the bank command and changes lateral position.

  The bank command is updated every $T = 50\ ms$. T is termed as the sampling period.

Digital Control Basics

- Two additional inputs to the system.
  - wind input $w(t)$ - disturbance that affects aircraft position.
  - radar noise $n(t)$ - sensor noise.

  Noise and disturbance are unwanted inputs but always present in control systems.

- The design problem is to maintain $x(t)$ small in the presence of noise and disturbances. Additionally, the system response should be acceptable.

Digital Control Basics

- How to go about designing the controller (i.e. solving the control problem)?

  We must know something about the system. Mathematical relationships between bank command $\phi(t)$, wind input $w(t)$ and the lateral position $x(t)$ would be helpful.

- Mathematical relationships are referred to as a mathematical model of the plant.

Digital Control Basics

- For an F4 aircraft, the lateral system model is a ninth-order ordinary nonlinear differential equation. For small $\phi(t)$, a linear model would still be a ninth-order ODE.

- The system design will be realized using a digital controller.

  The controller should account for the ninth-order ODE model, the wind disturbance, radar noise, the sampling period and the desired system response.

The Control Problem

- A physical system is to be accurately controlled by means of a closed-loop feedback system.

  The output (system response) is adjusted based on the error signal.

  The error is the difference between the actual and desired system responses.

- The controller processes the error signal based on several general criteria such as
  - steady-state error
  - transient response
  - disturbance rejection
  - sensitivity to plant changes

The Control Problem

- To solve the control problem will involve
  - choosing sensors to measure feedback signals.
  - choosing actuators to drive the plant.
  - developing plant (and also sensor and actuator) models.
  - designing the controller based on models and the performance criteria.
  - evaluating the design (analytically, simulation and physical tests).
  - iterating until satisfied with system response.

The Control Problem

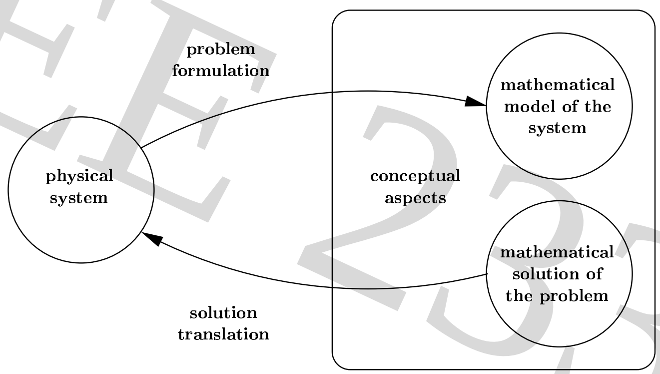
- Iteration. Actual physical tests may not yield satisfactory performance. Go back and redesign to improve the performance.

- Intuition. While experimenting with a physical system, designers usually develop an intuition on how to achieve the requirements.

- Experience. Depending on the design engineer's experience, some steps in the design may be skipped.

The Control Problem

- Approach. Mathematical analysis and design.



The Control Problem

- Given a physical system and the control problem,
  - pose the problem as a mathematical problem.
  - come up with mathematical solution.
  - translate the mathematical solution into reality.

- We need mathematical models to get going.

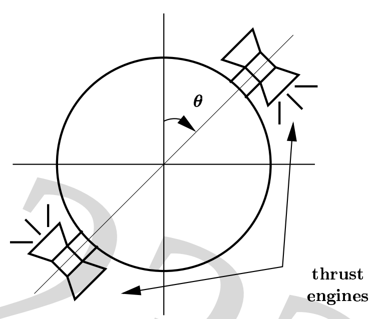
  Then, we apply our vast mathematical experience to come up with an equation describing the solution.

  Finally, we need to be build it.

Mathematical Models

- Example. Satellite model.

  Assume that the satellite is spherical and is controlled by thrust engines.



- Independent thrust control along the roll, pitch and yaw axes.

Mathematical Models

- Thrust engines develop torque to rotate the satellite on one axis.

  Satellite is rigid and space is a frictionless environment.

- Differential equation model.

$$J_r \frac{d^2\theta(t)}{dt^2} = \tau(t)$$

  where J_r is the satellite's moment of inertia about the roll axis and θ is the roll angle.

- Transfer function model.
  Let $\mathscr{L}[\tau(t)] = T(s)$ and $\mathscr{L}[\theta(t)] = \Theta(s)$.

$$J_r s^2 \Theta(s) = T(s)$$

$$G_r(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{J_r s^2}$$

- State-space model.
  Let $x_1(t) = \theta(t)$ and $x_2(t) = \dot{x}_1(t) = \dot{\theta}(t)$.

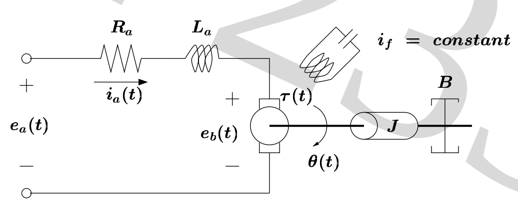
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_r} \end{bmatrix} \tau(t)$$

- Thus, the satellite model can be specified either by a second-order ODE, a transfer function or in state-space form.

  What to use? Depends on the stage of the design process.

- Example. Servomotor system model.
  - uses an electric motor for position control.
  - applications in antenna pointing system and manipulator arm control.

- Armature controlled DC motor with constant field.



- Assuming that the armature inductance L_a is small, we have four electromechanical equations

$$e_a(t) = R_a i_a(t) + e_b(t) \quad \text{(KVL)}$$

$$e_b(t) = k_b \frac{d\theta(t)}{dt} \quad \text{(back-EMF)}$$

$$\tau(t) = k_i i_a(t) \quad \text{(motor torque)}$$

$$\tau(t) = J\frac{d^2\theta(t)}{dt} + B\frac{d\theta(t)}{dt} \quad \text{(sum torques)}$$

- Solve the equations for the output $\theta(t)$ in terms of the input $e_a(t)$.

Mathematical Models

- From the KVL and back-EMF equations, we get

$$i_a(t) = \frac{e_a(t) - e_b(t)}{R_a} = \frac{e_a(t)}{R_a} - \frac{k_b}{R_a}\frac{d\theta(t)}{dt}$$

- Using the torque equations,

$$\tau(t) = k_i i(t) = \frac{k_i}{R_a}e(t) - \frac{k_i k_b}{R_a}\frac{d\theta(t)}{dt}$$
$$= J\frac{d^2\theta(t)}{dt} + B\frac{d\theta(t)}{dt}$$

Mathematical Models

- Second-order differential equation model.

$$J\frac{d^2\theta(t)}{dt} + \frac{BR_a + k_i k_b}{R_a}\frac{d\theta(t)}{dt} = \frac{k_i}{R_a}e_a(t)$$

- Laplace transform and transfer function.

$$G(s) = \frac{\Theta(s)}{E_a(s)} = \frac{\frac{k_i}{R_a}}{Js^2 + \frac{BR_a + k_i k_b}{R_a}s} = \frac{\frac{k_i}{JR_a}}{s\left(s + \frac{BR_a + k_i k_b}{JR_a}\right)}$$

Mathematical Models

- State-space representation.
  Let $x_1(t) = \theta(t)$ and $x_2(t) = \dot{x}_1(t) = \dot{\theta}(t)$.

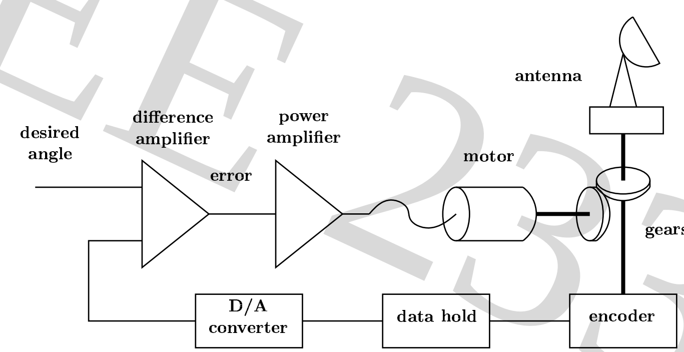
$$\dot{x}_2(t) = \ddot{\theta}(t) = -\frac{BR_a + k_i k_b}{JR_a}x_2(t) + \frac{k_i}{JR_a}e_a(t)$$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{BR_a + k_i k_b}{JR_a}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{k_i}{JR_a}\end{bmatrix}e_a(t)$$

- Note : if the armature inductance L_a is not negligible, the system model will be third-order.

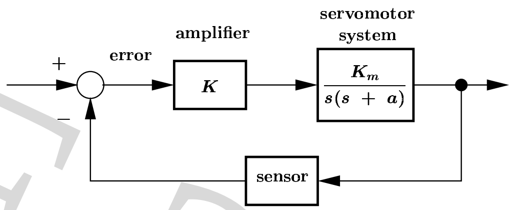
Mathematical Models

- Servomechanism application.
  Antenna pointing system (yaw angle control).



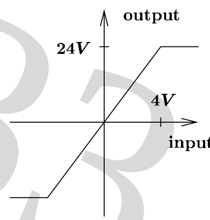
Mathematical Models

- Block diagram of the antenna pointing system.



- Error is a low power signal. An amplifier is used to drive the servomotor.

  The amplifier introduces a nonlinearity to the control system.



Mathematical Models

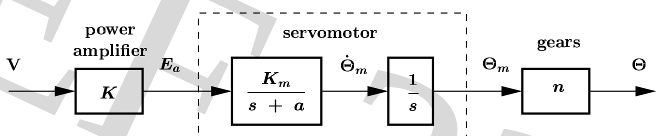
- Manipulator arm servomechanisms.
  - control the position (angles) of the joints of a manipulator.
  - manipulator dynamics are usually nonlinear differential equations.

- Simple approach.

  Consider the arm as a simple mechanism and ignore the effect of the motion of other arms.
  ⇒ simple but usually yields unacceptable response.

Mathematical Models

- The arm (link) is connected to the motor through gears at coupled at the joint.



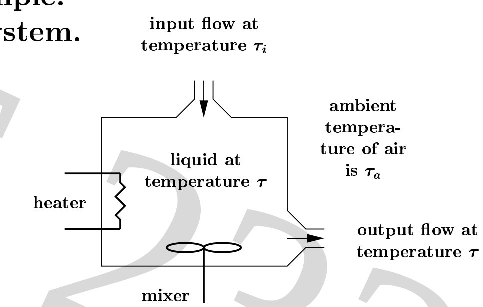
- Second-order model for the servomotor is assumed.

  If the gears turn ratio $n = \theta/\theta_m \ll 1$, then the inertia of the arm as seen by the motor will be small (and friction may be neglible).

Mathematical Models

- Another modeling example. Temperature control system.

  Liquid is flowing out at a certain rate while being replaced by liquid with temperature τ_i.



- The liquid is heated by an electric heater and agitated by a mixer such that the liquid temperature is uniform inside the tank.

- Define

$q_e(t) \triangleq$ heat increase supplied by the heater.
$q_i(t) \triangleq$ heat increase from entering liquid.
$q_l(t) \triangleq$ heat absorbed by the liquid.
$q_o(t) \triangleq$ heat decrease from exiting liquid.
$q_s(t) \triangleq$ heat loss through tank surface.

- Conservation of energy.

$$q_i(t) + q_e(t) = q_l(t) + q_o(t) + q_s(t)$$

- The heat absorbed by the liquid is related to the thermal capacity C and the temperature change by

$$q_l(t) = C\frac{d\tau(t)}{dt}$$

- Let $v(t)$ be the liquid flow rate in (and out) of the tank. If H is the specific heat of the liquid, then

$$q_i(t) = v(t)H\tau_i(t) \text{ and } q_o(t) = v(t)H\tau(t)$$

- The heat loss through the tank surface in terms of the thermal resistance R of the tank surface is

$$q_s(t) = \frac{\tau(t) - \tau_a(t)}{R}$$

- Combining the above equations, we get the differential equation model of the system.

$$q_e(t) + v(t)H\tau_i(t) = C\frac{d\tau(t)}{dt} + v(t)H\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

- Assuming that the flow rate $v(t)$ is at a constant value V, we get a time-invariant first-order ODE.

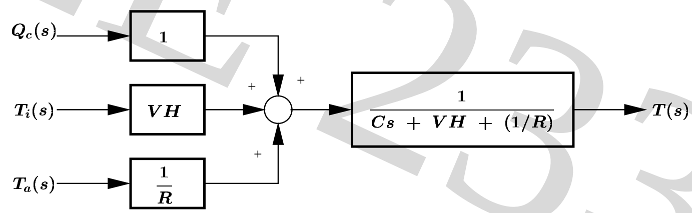
$$q_e(t) + VH\tau_i(t) = C\frac{d\tau(t)}{dt} + VH\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

- In view of a control system,
  - $q_e(t)$ is the input,
  - $\tau_i(t)$ and $\tau_a(t)$ are disturbances, and
  - $\tau(t)$ as the output.

Mathematical Models

- The transfer function and corresponding block diagram are

$$T(s) = \frac{Q_c(s) + VHT_i(s) + (1/R)T_a(s)}{Cs + VH + (1/R)}$$



Mathematical Models

- We can consider the system to have multiple inputs. If the disturbance inputs are ignored, then the system is a simple first-order system.

  This may be true if we have almost no input flow and the tank is well insulated.

- Although the design would be simple, it may not perform well is reality.
  Disturbance rejection should also be accounted for when designing a system.

Summary

- We have reviewed the concept of a closed-loop system.
- Discussed three physical systems and introduce the concept of digital control.
- Developed some models for physical systems.
- Review of math basics and different model formulations.