EE 233 Homework 4.

3-2. A signal $e(t)$ is sampled by an ideal sampler.

a. List the conditions under which $e(t)$ can be completely recovered from $e^*(t)$, i.e., the conditions under which no loss of information by the sampling process occurs.

Solution:

1. $e(t)$ should not have any frequency components above $\omega_s/2$.
2. implement an ideal low pass filter.

b. State which of the conditions listed in a. can occur in a physical system. Recall that the sampling operation itself is not physically realizable.

Solution:

First condition only.

c. Considering the answers in b., state why we can successfully employ systems that use sampling.

Solution:

We can come up with very good approximations to an ideal low pass filter.

3-8. Find $E^*(s)$ for

$$E(s) = \frac{1 - e^{-Ts}}{s(s + 1)}$$

Solution:
Let

$$E_1(s) = \frac{1}{s(s + 1)}$$

$$E_1^*(s) = \sum_{\substack{\lambda = 0 \\ \lambda = -1}} \left[\text{residues of } \frac{1}{\lambda(\lambda + 1)\left[1 - \epsilon^{-(s - \lambda)T}\right]} \right]$$

$$= \frac{1}{1 - \epsilon^{-Ts}} - \frac{1}{1 - \epsilon^{-T(1 + s)}}$$

Thus,

$$
\begin{aligned}
E^*(s) &= E_1^*(s) - \epsilon^{-Ts} E_1^*(s) \\
&= \left(1 - \epsilon^{-Ts}\right)\left(\frac{1}{1 - \epsilon^{-Ts}} - \frac{1}{1 - \epsilon^{-T(1 + s)}}\right) \\
&= 1 - \frac{1 - \epsilon^{-Ts}}{1 - \epsilon^{-T(1 + s)}}
\end{aligned}
$$

3-14.

a. A sinusoid with a frequency of 2 Hz is applied to a sampler/zero-order hold combination. The sampling rate is 10 Hz. List all frequencies present in the output that are less than 50 Hz.

Solution: { 2, 8, 12, 18, 22, 28, 32, 38, 42, 48 } Hz.

b. Repeat a. if the input sinusoid has a frequency of 8 Hz.

Solution: { 8, 2, 18, 12, 28, 22, 38, 32, 48, 42 } Hz.

c. The results of a. and b. are identical. Give three other frequencies, which are greater than 50 Hz, that yield the same results as a. and b.

Solution: { 52, 58, 62 } Hz.