EE 233 Homework 2.

2-3. Find the z-transforms of the number sequences generated by sampling the following time functions every T seconds, beginning at $t = 0$. Expressed the transforms in closed-form.

a. $e(t) = e^{-at}$

Solution:

$$\begin{aligned} E(z) &= 1 + e^{-aT} z^{-1} + e^{-2aT} z^{-2} + \ldots \\ &= 1 + e^{-aT} z^{-1} + \left(e^{-aT} z^{-1}\right)^2 + \ldots \\ &= \frac{1}{1 - e^{-aT} z^{-1}} = \frac{z}{z - e^{-aT}} \end{aligned}$$

b. $e(t) = e^{-(t - T)} u(t - T)$

Solution:

$$\begin{aligned} E(z) &= z^{-1} + e^{-T} z^{-2} + \ldots \\ &= z^{-1} \left(1 + e^{-T} z^{-1} + \ldots\right) \\ &= z^{-1} \frac{z}{z - e^{-T}} = \frac{1}{z - e^{-T}} \end{aligned}$$

c. $e(t) = e^{-(t - 5T)} u(t - 5T)$

Solution:

$$\begin{aligned} E(z) &= z^{-5} + e^{-T} z^{-6} + \ldots \\ &= z^{-5} \left(1 + e^{-T} z^{-1} + \ldots\right) \\ &= z^{-5} \frac{z}{z - e^{-T}} = \frac{z^{-4}}{z - e^{-T}} \end{aligned}$$

2-4. Find the z-transform, in closed-form, of the number sequence generated by sampling the time function $e(t)$ every T seconds beginning at $t = 0$. The Laplace transform of $e(t)$ is

$$E(s) = \frac{2\left(1 - e^{-5s}\right)}{s(s + 2)}, \quad T = 1 \text{ s}$$

Solution:

$$\begin{aligned} e(t) &= 1 - e^{-2t} - \left[1 - e^{-2(t - 5)}\right] u(t - 5) \\ E(z) &= \left[\frac{z}{z - 1} - \frac{z}{z - e^{-2T}}\right] \left(1 - z^{-5}\right) \end{aligned}$$

2-8. FInd the inverse z-transform of each $E(z)$ below by using the four methods described in the lecture (power series, partial fraction expansion, inversion, and discrete convolution). Compare the values of $e(k)$ for $k = 0, 1, 2, 3$, obtained by the four methods.

a.

$$E(z) = \frac{0.5z}{(z - 1)(z - 0.6)}$$

Solution:

$$\frac{E(z)}{z} = \frac{0.5}{(z - 1)(z - 0.6)} = \frac{1.25}{z-1} - \frac{1.25}{z - 0.6}$$

$$e(k) = 1.25\left(1 - 0.6^k\right)$$

b.

$$E(z) = \frac{0.5}{(z - 1)(z - 0.6)}$$

Solution:

$$e(k) = 1.25\left[1 - 0.6^{(k - 1)}\right] u(k - 1)$$

c.

$$E(z) = \frac{0.5(z + 1)}{(z - 1)(z - 0.6)}$$

Solution:

$$e(k) = 1.25\left\{1 - 0.6^k + \left[1 - 0.6^{(k - 1)}\right] u(k - 1)\right\}$$

d.

$$E(z) = \frac{z(z - 0.7)}{(z - 1)(z - 0.6)}$$