EE 233 Homework 1.

1-10. A thermal test chamber is illustrated in Figure 1(a). This chamber, which is a large room, is used to test large devices under various thermal stresses. The chamber is heated with steam, which ic controlled by an electrically activated valve. The temperature of the chamber is measured by a sensor based on a thermistor. Opening the door into the chamber affects the chamber temperature and thus must be considered as a disturbance.

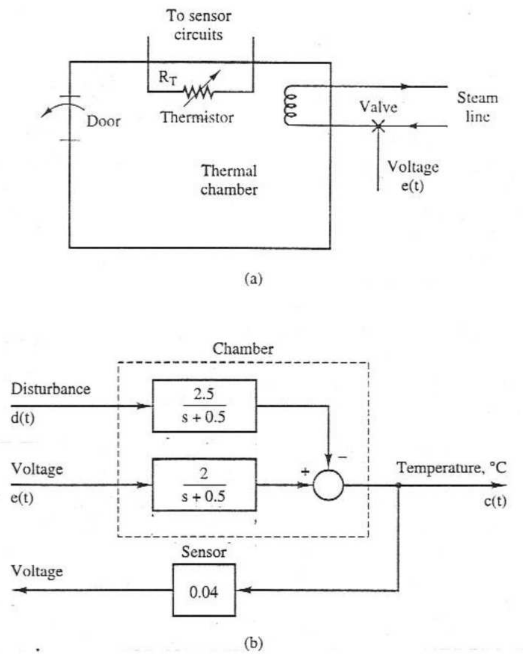


Figure 1:

A simplified model of the test chamber is shown in Figure 1(b), with time units is minutes. The control input is voltage $e(t)$, which controls the valve in the steam line. The disturbance $d(t)$, a unit step function is used to model the door. With the door closed, $d(t) = 0$.

a. Find the time constant of the chamber.
b. With the voltage $e(t) = 5u(t)$ and the chamber door closed, find and plot the chamber temperature $c(t)$. Also find the steady-state temperature c_{ss}.
c. In part a., it is assumed that the initial chamber temperature of 0 ^oC. Repeat part b., assuming that the initial temperature is $c(0) = 25\ ^oC$.
d. Two minutes after the application of the voltage in part c., the door is opened, and it remains open. Find and plot the chamber temperature $c(t)$ considering the effect of the disturbance.
e. The door is part d., remains open for 12 minutes and is then closed. Find the new chamber temperature $c(t)$ consideing the effect of the new disturbance.

Solution:

a. With $d(t) = 0$, chamber transfer function is

$$G(s) = \frac{2}{s + 0.5}$$

The pole is at $s = -0.5$, thus $\tau = 1/0.5 = 2\ min$.

b. $e(t) = 5u(t)$, $E(s) = 5/s$.

$$C(s) = G(s)E(s) = \frac{2}{s + 0.5} \cdot \frac{5}{s} = \frac{-20}{s + 0.5} + \frac{20}{s}$$

$$\begin{aligned} c(t) &= 20 - 20e^{-0.5t} \\ c_{ss} &= 20\ ^oC \end{aligned}$$

c. by superposition, with $c(0) = 25\ ^oC$,

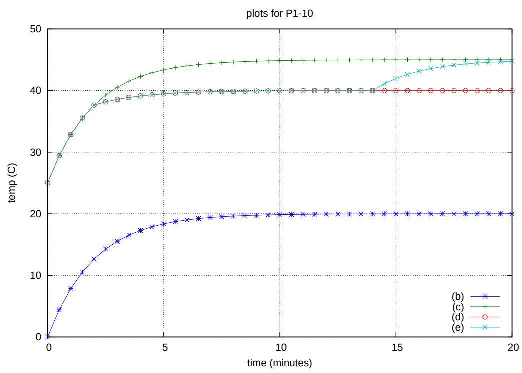
$$\begin{aligned} c(t) &= 25 + 20 - 20e^{-0.5t} \\ c(t) &= 45 - 20e^{-0.5t} \end{aligned}$$

d. $d(t) = u(t - 2)$, define $c_d(t)$ be the contribution of the disturbance to $c(t)$.

$$\begin{aligned} c_d(t) &= 5\left[1 - e^{-0.5(t - 2)}\right] u(t - 2) \\ c(t) &= 45 - 20e^{-0.5t} - 5\left[1 - e^{-0.5(t - 2)}\right] u(t - 2) \end{aligned}$$

e. with the door open for 12 minutes, we have $d(t) = u(t - 2) - u(t - 14)$.

$$\begin{aligned} c_d(t) &= 5\left[1 - e^{-0.5(t - 2)}\right] u(t - 2) - 5\left[1 - e^{-0.5(t - 14)}\right] u(t - 14) \\ c(t) &= 45 - 20e^{-0.5t} - 5\left[1 - e^{-0.5(t - 2)}\right] u(t - 2) + 5\left[1 - e^{-0.5(t - 14)}\right] u(t - 14) \end{aligned}$$



1-14. The input to the satellite system in Figure 2 is a step function $\theta_c(t) = 5u(t)$ in degrees. As a result, the satellite angle $\theta(t)$ varies sinusoidally at a frequency of 10 cycles per minute. Find the amplifier gain K and the moment of inertia J for the system, assuming that the units of time in the system equations are seconds.

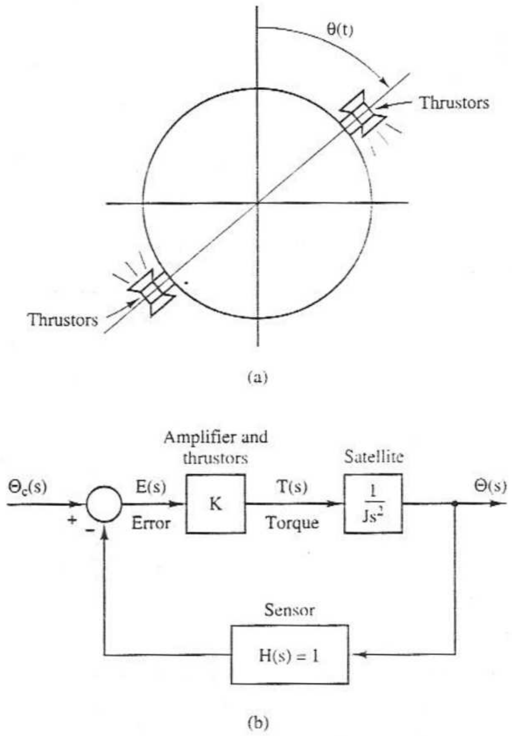


Figure 2:

Solution:

$$G(s) = \frac{K/Js^2}{1 + K/Js^2} = \frac{K/J}{s^2 + K/J}$$

$$\Theta(s) = \frac{K/J}{s^2 + K/J} \cdot \frac{5}{s}$$

Since $\theta(t)$ is 10 cycles per minute,

$$f = \frac{10}{min} = \frac{1}{6}\,Hz$$

$$\omega = 2\pi f = \frac{\pi}{3}\,\frac{rad}{s}$$

Also, from the TF, poles are at

$$
\begin{aligned}
poles &= \pm\sqrt{K/J} = \frac{\pi}{3} \\
\frac{K}{J} &= \left[\frac{\pi}{3}\right]^2
\end{aligned}
$$

1-15. The satellite control system of 1-14 is not usable since the response to any excitation includes an undamped sinusoid. The usual compensation for this system involves measuring the angular velocity $d\theta/dt$. The feedback signal is then a linear sum of the position signal $\theta(t)$ and the velocity signal $d\theta(t)/dt$. The sysem is depicted in Figure 3 and is said to have rate feedback.

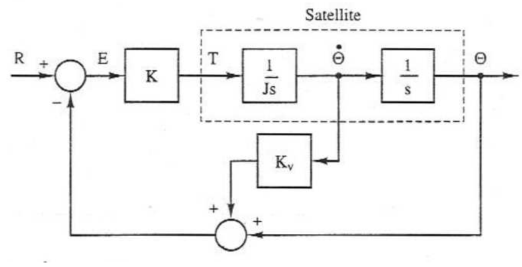


Figure 3:

a. Derive the transfer function $\Theta(s)/\Theta_c(s)$ for the system.
b. Derive the state equations for the system.

Solution:

a. Writing the equation for the error variable.

$$E = R + EK\frac{1}{Js}K_v + EK\frac{1}{Js^2}$$

$$R = E\left[1 - \frac{K}{Js}\left(K_v + \frac{1}{s}\right)\right]$$

Also,

$$\Theta(s) = E(s)K\frac{1}{Js^2}$$

$$\Theta_c(s) = R(s)$$

Thus, the transfer function is

$$\frac{\Theta(s)}{\Theta_c(s)} = \frac{K/Js^2}{1 - \frac{K}{Js}\left(K_v + \frac{1}{s}\right)} = \frac{K}{Js^2 - K(K_v s + 1)}$$

b. Derive the state equations for the system.

Rewriting the transfer function equation above,

$$Js^2\Theta(s) \; - \; KK_v s\Theta(s) \; + \; K\Theta(s) \; = \; \Theta_c(s)$$

Taking the inverse Laplace and isolating $\ddot{\theta}$,

$$J\ddot{\theta} \; - \; -KK_v\dot{\theta} \; - \; K\theta \; = \; K\theta_c$$

$$\ddot{\theta} \; = \; \frac{1}{J}\left[KK_v\dot{\theta} \; + \; K\theta\right] \; + \; \frac{K}{J}\theta_c$$

Assigning the state variable $x \; = \; [\theta \; \dot{\theta}]^T$,

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \; = \; \begin{bmatrix} 0 & 1 \\ K/J & KK_v/J \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \; + \; \begin{bmatrix} 0 \\ K/J \end{bmatrix} \theta_c$$