Today's EE 231 Lecture

- Identifying transfer functions.

  Why use Bode plots to identify transfer functions?

- Some performance parameters.

- Compensation techniques.

- Interpreting Bode plots.
  - low frequency response.
  - high frequency response.

Identifying Transfer Functions

- How do we identify the transfer function for unknown systems?
  - step (time domain) response.
  - frequency response (Bode plots).

- Step response method.
  - apply a unit step at the input.
  - measure rise time, delay time, steady-state output and other time domain parameters.
  - infer the transfer function from the measurements.

Identifying Transfer Functions

- Frequency response method.
  - apply a sinusoidal input and sweep the frequency ω from low frequencies to high frequencies.
  - plot the logarithmic gain and phase plots.
  - identify system type, gain, poles and zeros.
  - determine the transfer function.

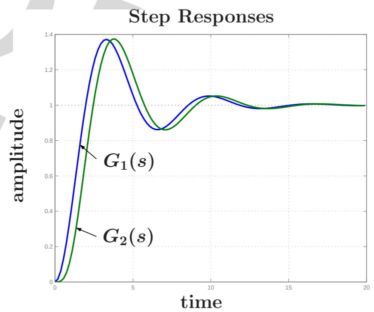
- Considerations on what method to use.
  - accuracy of identification.
  - system response (fast or slow) of the plant.

Identifying Transfer Functions

- Recall the example comparing

$$G_1(s) = \frac{1}{s^2 + 0.6s + 1} \text{ and}$$
$$G_2(s) = G_1(s)\frac{9}{s^2 + 4.2s + 9}$$

Not much difference in the step responses of $G_1(s)$ and $G_2(s)$.

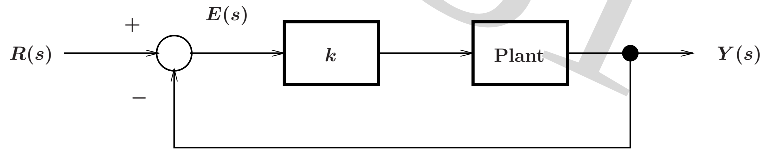


Identifying Transfer Functions

- Thus, may not be able to accurately determine the transfer function $G_2(s)$ from its step response.

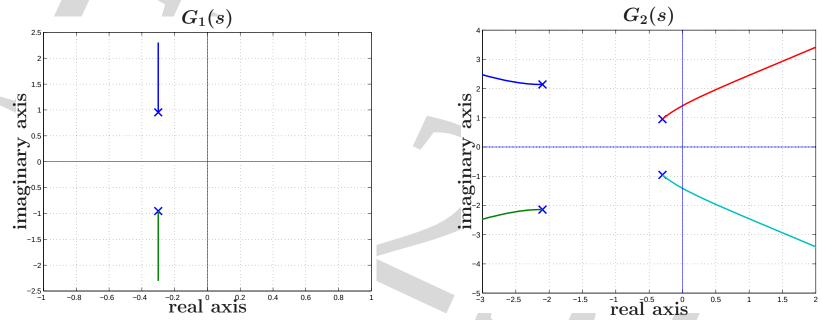
  Given the step response of $G_2(s)$, one might incorrectly identify the TF as that of $G_1(s)$.

- Controller designed for $G_1(s)$ may not necessarily work for $G_2(s)$. Consider



Identifying Transfer Functions

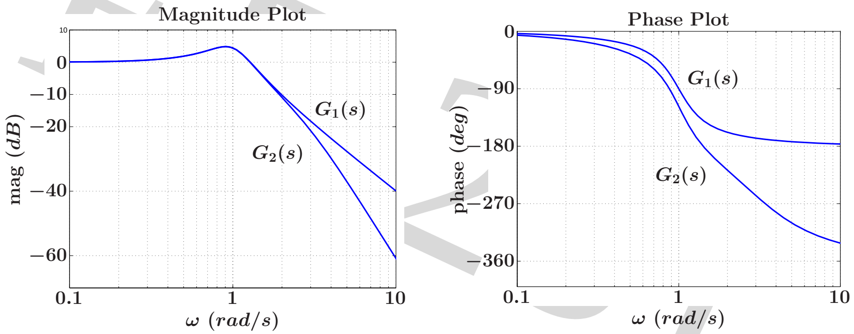
- Root loci of $G_1(s)$ and $G_2(s)$.



plant $G_1(s)$: the system is stable for any gain k.
plant $G_2(s)$: the system is unstable for gain $k > 1.35$.

Identifying Transfer Functions

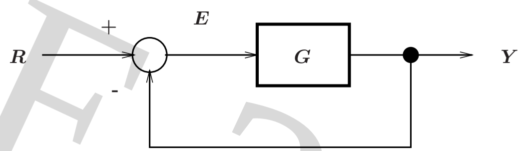
- If we use the frequency response to identify the TF, cannot mistake $G_1(s)$ and $G_2(s)$ for the other.



Compare the slopes of the magnitude plot and the asymptotic values of the phase angles for large ω.

Performance Parameters

- Effect on the steady-state error, e_{ss} for a unit step input.



$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)}$$

For a step input : $r(t) = u(t)$, then

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1 + G(s)} \frac{1}{s} = \frac{1}{1 + G(0)}$$

- Assuming $G(0) > 0$, increasing $G(0)$ decreases the steady-state error.

- From the Bode plots, the logarithmic gain at $\omega = 0$ is

$$[20 \log |G(j\omega)|]_{\omega=0} = 20 \log |G(0)|$$

For a system of type 0, $20 \log |G(j0)|$ corresponds to the horizontal asymptote at low frequencies.

$\Rightarrow$ to decrease the steady-state error e_{ss}, move the low frequency horizontal asymptote upwards.

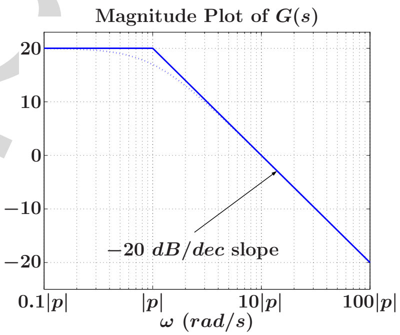
- Effect on the rise time.
  Recall for the first order system $G(s) = \dfrac{a}{s + a}$,
  the rise time T_r is approximately given by $T_r = \dfrac{2.2}{a}$

- From the Bode plots of a pole on the real axis, $|p| = |-a|$ is the corner frequency.

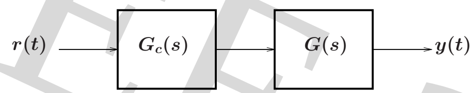
- Thus, to decrease the rise time (faster system response), increase a. Move the corner frequency $|p|$ to the right.

- Consider $G(s) = \dfrac{k}{s - p}$.

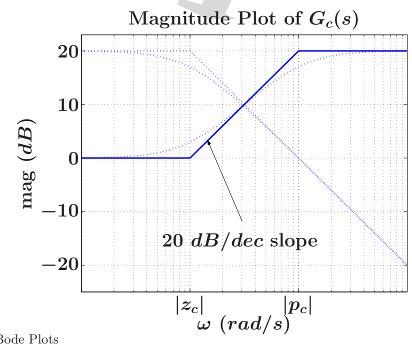
- To move the location of the corner frequency,
  - introduce a zero at location z_c such that $z_c = p$.
  - add the necessary pole at location p_c to achieved the desired response.
  - add gain.

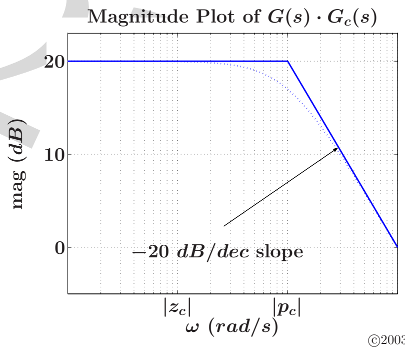


- Use the controller $G_c(s)$.



$$G_c(s) = k_c \frac{s - z_c}{s - p_c}$$





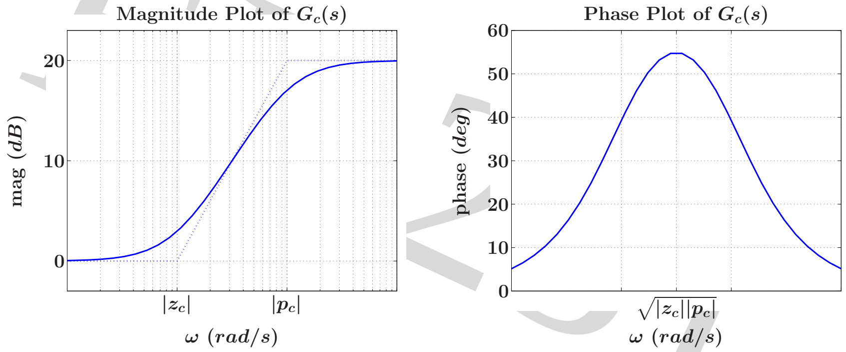
- Phase-lead and phase-lag controller.

$$G_c(s) = k_c \frac{s - z_c}{s - p_c}$$

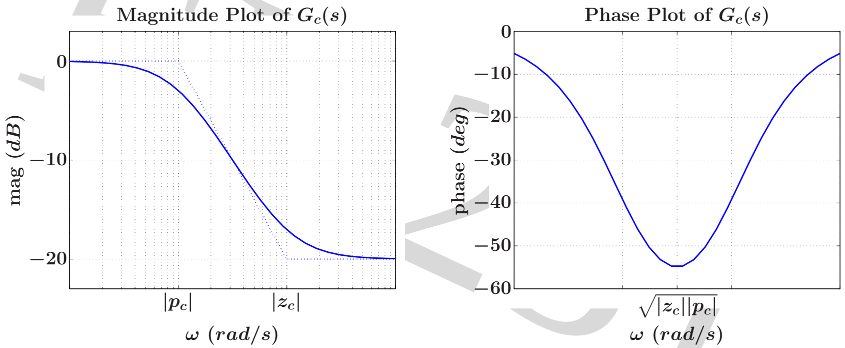
phase-lead : $|z_c| \ll |p_c|$
phase-lag : $|p_c| \ll |z_c|$

- Design considerations (for phase-lead controller).
  - z_c is located at the pole location of the original TF.
  - p_c is determined based on performance specifications.
  - k_c is computed such that there is no upward shift in the magnitude plot, i.e., $G(0) \cdot G_c(0) = G(0)$.

- Phase-lead controller Bode plots.



- Phase-lag controller Bode plots.



- Effect of gain factor is constant at all frequencies.
  - magnitude plot : $20 \log |k|$
  - phase plot : 0^o for $k > 0$ or -180^o for $k < 0$

- Slope of magnitude plot depends on integral and derivative factors.

  Let N be the difference in the number of poles and the number of zeros at the origin.
  - magnitude plot : slope $= -20N \; dB/dec$
  - phase plot : asymptote $= -90N \; degrees$

Interpreting Bode Plots : Low Frequency

- **Steady-state error.** Represent the value of the magnitude plot at $w = 1\ rad/s$ as M.
  - type 0 : $20\log|k_p| = M \Rightarrow k_p = 10^{M/20}$
$$e_{ss} = \frac{1}{1 + k_p} \text{ for a unit step input}$$
  - type 1 : $20\log|k_v| = M \Rightarrow k_v = 10^{M/20}$
$$e_{ss} = \frac{1}{k_v} \text{ for a ramp input}$$
  - type 2 : $20\log|k_a| = M \Rightarrow k_a = 10^{M/20}$
$$e_{ss} = \frac{1}{k_a} \text{ for a parabolic input}$$

Interpreting Bode Plots : High Frequency

- Each pole eventually provides $-20\ dB/dec$ magnitude slope and -90^o phase shift.

  Each zero eventually provides $20\ dB/dec$ magnitude slope and 90^o phase shift.

  With n = number of poles and m = number of zeros,
  - magnitude plot : slope = $-20(n - m)\ dB/dec$
  - phase plot : asymptote = $-90(n - m)$ *degrees*

- Difference in the number of poles and the number of zeros is related to the stability of the closed-loop system.

Summary of Today's Lecture

- Identifying transfer functions.
  - using the step response.
  - using Bode plots.
- Some performance parameters.
  - steady-state error.
  - rise time.
- Phase-lead / phase-lag compensation techniques.
- Interpreting Bode plots.