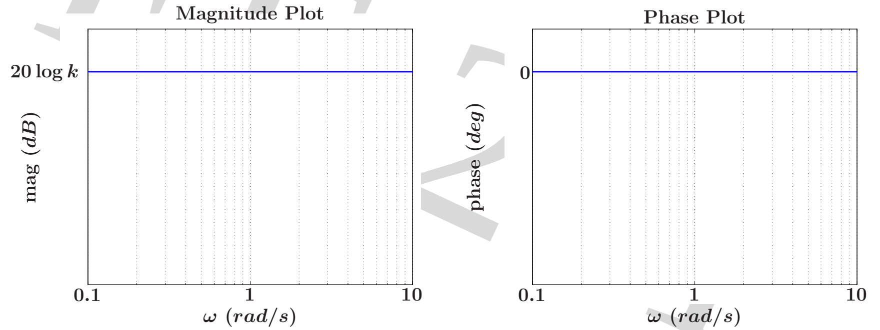
Today's EE 231 Lecture

- Review of standard Bode plots.
  - pure gain.
  - poles and zeros at the origin.
  - poles and zeros on the real axis.
  - complex conjugate pairs poles / zeros.
- Building an asymptotic Bode plot.
- Identifying a transfer function from a Bode plot.
- Summary.

Standard Bode Plots

- Pure gain.

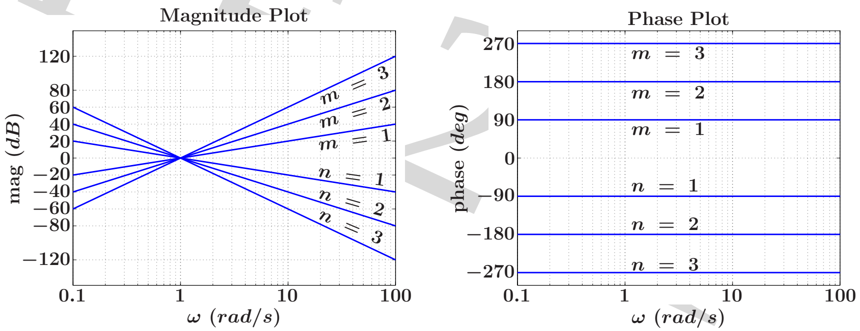
$$k > 0$$



Standard Bode Plots

- Poles and zeros at the origin.

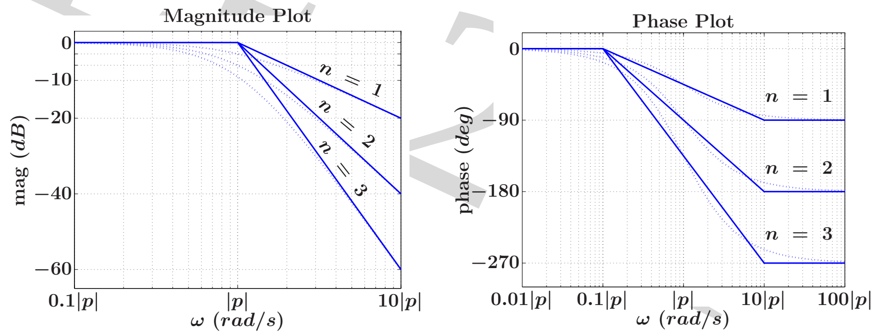
$$\text{poles}: \frac{1}{s^n} \qquad \text{zeros}: s^m$$



Standard Bode Plots

- Poles on the real axis.

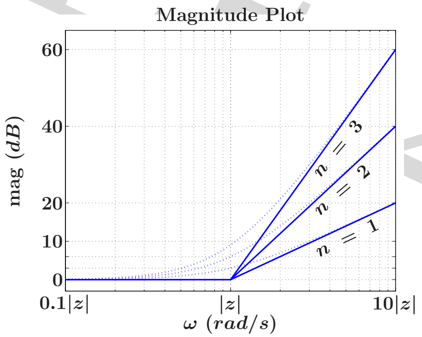
$$\frac{(-p)^n}{(s - p)^n}, \qquad p < 0$$

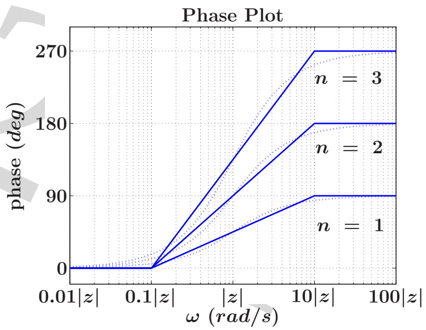


Standard Bode Plots

- Zeros on the real axis.

$$\frac{(s - z)^n}{(-z)^n}, \qquad z < 0$$

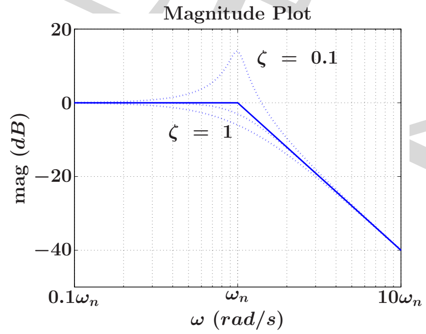


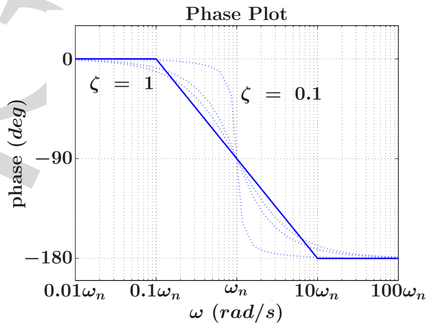


Standard Bode Plots

- Complex conjugate poles.

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad \omega_n > 0 \text{ and } 0 \le \zeta \le 1$$





Standard Bode Plots

- Example. Consider the following transfer function.

$$G(s) = \frac{100s^2\left(s + 10^2\right)\left(s + 10^6\right)^2}{(s + 10)^2\left(s + 10^3\right)\left(s + 10^5\right)^3}$$

- Moving some factors around, we have

$$G(s) = \frac{10^{16}}{10^{20}} \cdot \frac{s^2\left(\frac{s}{10^2} + 1\right)\left(\frac{s}{10^6} + 1\right)^2}{\left(\frac{s}{10} + 1\right)^2\left(\frac{s}{10^3} + 1\right)\left(\frac{s}{10^5} + 1\right)^3}$$

Standard Bode Plots

- Asymptotic magnitude plots.

Let

$k = 10^{-4}$

$z_1 = 0 \ (m2)$

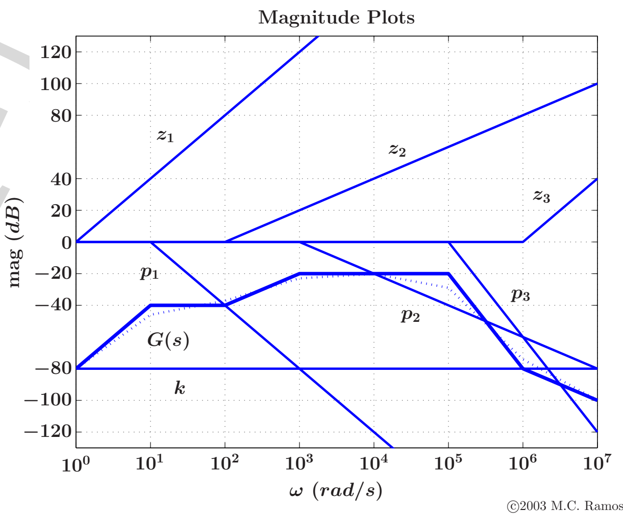
$z_2 = -10^2$

$z_3 = -10^6 \ (m2)$

$p_1 = -10 \ (m2)$

$p_2 = -10^3$

$p_3 = -10^5 \ (m3)$

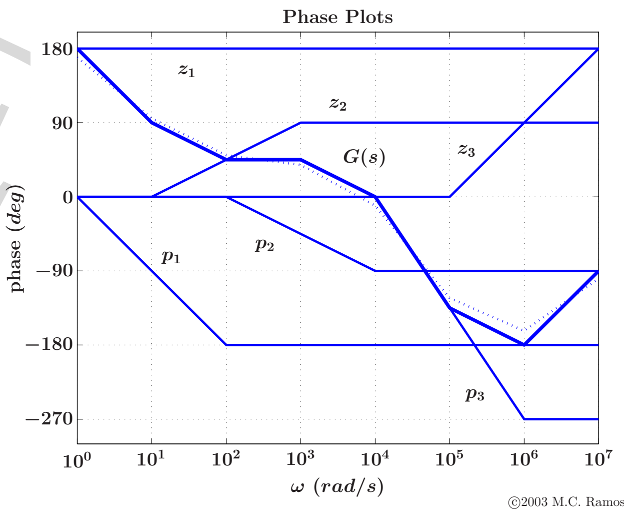


Standard Bode Plots

- Asymptotic phase plots.

Let

$k = 10^{-4}$
$z_1 = 0 \ (m2)$
$z_2 = -10^2$
$z_3 = -10^6 \ (m2)$
$p_1 = -10 \ (m2)$
$p_2 = -10^3$
$p_3 = -10^5 \ (m3)$



Identifying the TF from the Bode Plots

- Learn and familiarize with 'standard' Bode plots.

- Identify the system type.
  - look at the slope of the magnitude plot at $\omega = 1$.
  - look at the phase angle as for small w.

- Find the asymptotes and the corner frequencies.
  - find the transitions or bends in the Bode plots.
  - extend the asymptotes to determine the corner frequencies.

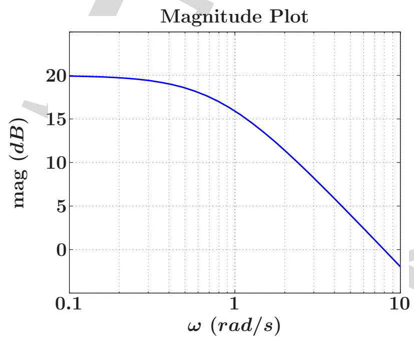
Identifying the TF from the Bode Plots

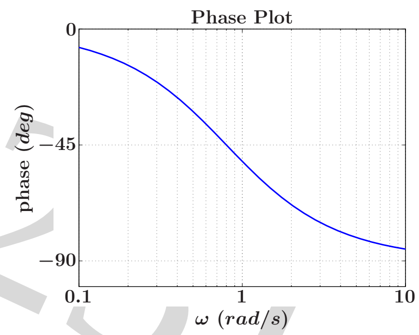
- Determine the general form of the transfer function.
  - trace the Bode plots and identify poles and zeros.
  - write out the general form and check.

- Compute for the system parameters.
  - use the magnitude plot to determine the gain, and the locations of poles and zeros.
  - use the phase plot to verify the locations, and possibly increase the accuracy.

Identifying the TF from the Bode Plots

- Example. Consider the following Bode plots.



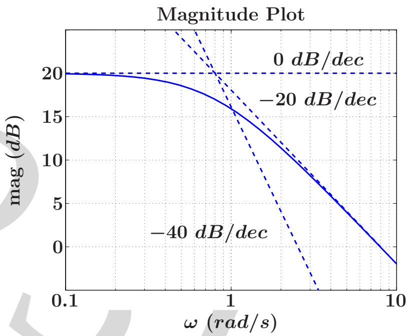


- Looking at the low frequencies,
  $\Rightarrow$ system type 0.

- What are the asymptotes?

At $\omega \rightarrow 0$, horizontal asymptote line at 20 dB.

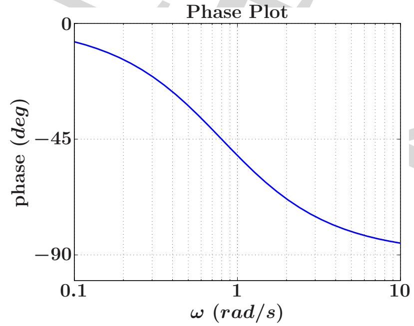
At large ω, $-20\ dB/dec$ asymptote line.



- The magnitude plot only has one transition (or bend). $\Rightarrow$ the transfer function has one pole.

- The general form of the TF is

$$G(s) = \frac{k}{s - p} \Rightarrow G(j\omega) = \frac{k}{j\omega - p}$$



Looking at $\omega \rightarrow 0$, we can see that $\phi(\omega) \rightarrow 0$.

Thus, assuming $k > 0$, the pole must be located in the LHP, i.e.,

$$p < 0$$

- We can see from the magnitude plot that the magnitude approaches 20 dB as $w \rightarrow 0$.

Using the general form of the transfer function for $\omega = 0$, the logarithmic gain is

$$20 \log \left| \frac{k}{-p} \right| = 20\ dB$$

Since $p < 0$ and $k > 0$,

$$\frac{k}{-p} = 10$$

- Using the general form, the phase angle at the corner frequency is

$$\phi(\omega) = -\tan^{-1}\left[\frac{\omega}{-p}\right] \Rightarrow \phi(\omega_c) = -\tan^{-1}\left[\frac{\omega_c}{-p}\right]$$

- From the standard Bode plots we know that corner frequency is located at $\omega_c = |p|$. Thus,

$$\phi(\omega_c) = -\tan^{-1}\left[\frac{\mp p}{-p}\right] = -45^o, \ -135^o$$

Use this equation to determine the location of ω_c (and consequently the pole p).

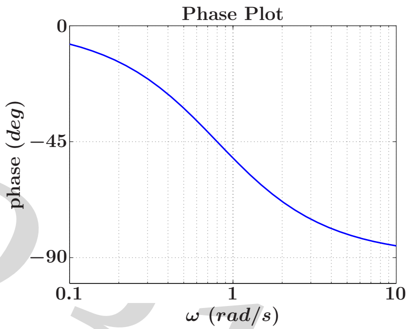
Identifying the TF from the Bode Plots

- From the plot we see that $\omega_c = 0.8\ rad/s$.

  Since $p < 0$ and $\omega_c = |p|$,

  $$p = -0.8$$

  Also, $k = (-p)(10) = 8$.



- Thus, the transfer function is

$$G(s) = \frac{8}{s - (-0.8)} = \frac{8}{s + 0.8}$$

Summary

- Learning typical Bode plots are important to
  - constructing the Bode plots for a given TF.
  - identifying the transfer function if the magnitude and phase plots are available.

- Why identify the transfer function from the Bode plots?

- Network analyzer does this automatically.