Today's EE 231 Lecture

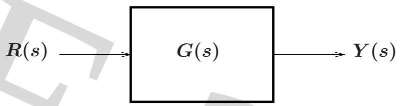
- Pole position and time domain relationships
- Typical time domain specifications
- First-order systems
- Second-order systems

Performance Specifications

- Typical time domain specifications
  - overshoot
  - settling time
  - rise time
  - time to peak
- Typical frequency domain specifications
  - natural frequency
  - damped frequency
  - damping ratio
  - bandwidth

Poles Affect Time Behavior

- Plant : $G(s)$



$$G(s) = k\frac{(s - z_1)\ \ldots\ (s - z_m)}{(s - p_1)\ \ldots\ (s - p_n)}$$

- Assuming no duplicated poles and $m < n$,

$$G(s) = \frac{R_1}{s - p_1} + \ \ldots\ + \frac{R_n}{s - p_n}$$

Poles Affect Time Behavior

- Impulse response.

$$\mathscr{L}^{-1} \Rightarrow g(t) = \left(R_1 e^{p_1 t} + \ \ldots\ + R_n e^{p_n t}\right) u(t)$$

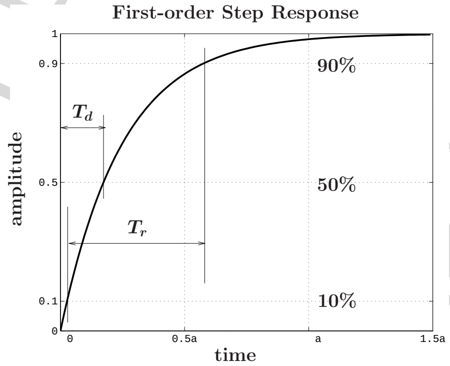
- Q : If systems are high-order, do we need to compute all the terms to get the response?

  A : Not always. High-order systems sometimes behave like low-order systems.

- Study low-order systems first.

- Consider

$$G(s) = \frac{a}{s + a}$$



- Q : What is the DC gain?

- Q : What is the steady-state output for $x(t) = u(t)$?

- Definitions.

rise time $\triangleq$ time for the step response
T_r to go from 10% to 90%.

delay time $\triangleq$ time for the step response to
T_d reach 50% of the final value.

- Relationships of T_r and T_d with system parameter a.

$$T_r = \frac{2.2}{a} = 2.2\tau \qquad T_d = \frac{0.69}{a} = 0.69\tau$$

- Octave command.

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>> step(2, [1 2])
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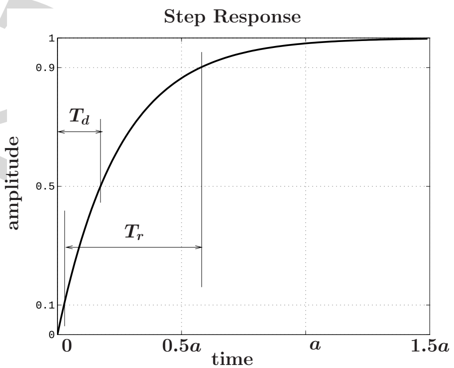
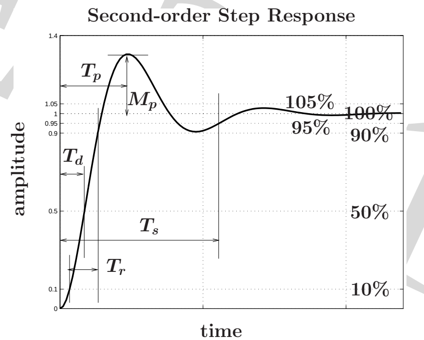


- Consider

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



- **What is the DC gain?**

  The DC gain is the steady-state output of the system in response to a step input.

- **Definitions.**

  $M_p \triangleq$ peak overshoot / max overshoot

  $T_p \triangleq$ time to peak overshoot

  $T_s \triangleq$ settling time

- **Poles of the second-order system.**

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

- **System parameters.**

  ω_n : natural frequency

  ζ : damping factor

- **Octave command.**

```
>> step(2, [1 1 2])
```


- **System characteristics.**

ζ	Description	Poles
$\zeta > 1$	real poles overdamped	LHP
$\zeta = 1$	real identical poles critically damped	LHP
$0 < \zeta < 1$	complex conjugate poles underdamped	LHP
$\zeta = 0$	imaginary poles undamped	imaginary axis
$\zeta < 0$	poles with positive real part negatively damped	RHP

- **Step response.**

$$Y(s) = G(s)\frac{1}{s} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

$$\Rightarrow y(t) = 1 - \frac{e^{-\frac{t}{\tau}}}{\sqrt{1-\zeta^2}} cos(\omega_d t - \rho_d),\ t \geq 0$$

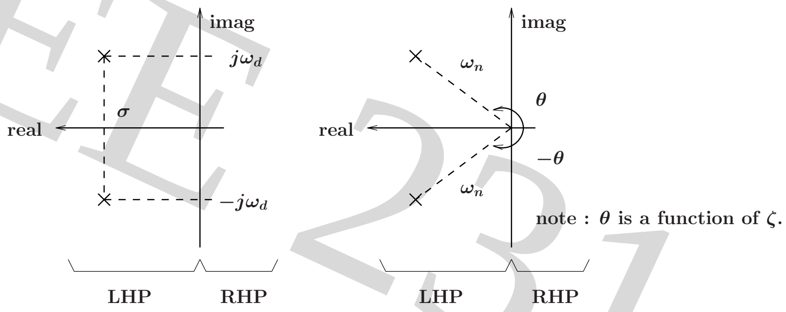
where

$$\sigma = -\zeta\omega_n \qquad \tau = \frac{1}{|\sigma|}$$

$$\rho_d = \sin^{-1}\zeta \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \leftarrow \text{damped } \omega$$

Second-order Systems

- Location of poles in the s-plane.



In Summary

- System parameters are the starting point in the design.
- The behavior of first-order systems is dictated by one parameter.
- Second-order systems behavior is governed by two parameters.
- Two ways of looking at the poles in the s-plane.