Where are we? Where are we heading?

- What have we done so far?
  - we are now acquainted with control systems.
  - we know what a model is and how to get it.
  - we have the tools to deal with the math.

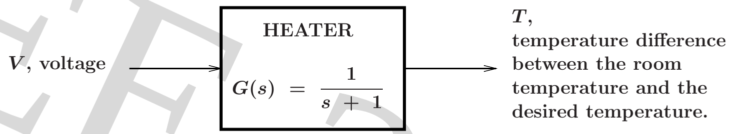
- What will be looking at next?
  - we will start investigating systems in detail.
  - we will work first with steady-state.
  - time domain and frequency domain characterizations.

- For today, look at system type and steady-state error.

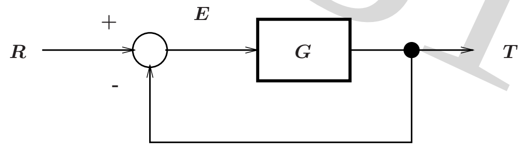
©2002 M.C. Ramos
UP EEE Institute

System Type

- Simple heater model (first-order, type 0).



- Add feedback.



©2002 M.C. Ramos
UP EEE Institute

System Type

- Steady-state error.

$$\frac{E}{R} = \frac{R - T}{R} = \frac{1}{1 + G} = \frac{s + 1}{s + 2}$$

- For a step input : $r(t) = u(t)$, then

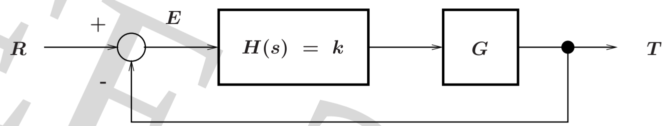
$$e_{ss} = \lim_{s \to 0} s \frac{s + 1}{s + 2} \frac{1}{s} = \frac{1}{2}$$

- For a type 0 system.
  $\Rightarrow$ non-zero tracking error in response to a step input

©2002 M.C. Ramos
UP EEE Institute

System Type

- Insert additional gain.




```
>> [n,d] = cloop(5, [1 1], -1)  % gain k = 5
>> step(n, d)
```

- Steady-state error.

$$\frac{E}{R} = \frac{s + 1}{s + 1 + k}$$

©2002 M.C. Ramos
UP EEE Institute

EE 231

- For a step input : $r(t) = u(t)$, then

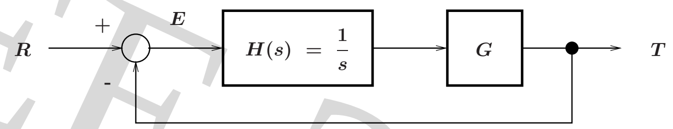
$$e_{ss} = \lim_{s \to 0} s \frac{s + 1}{s + 1 + k} \frac{1}{s} = \frac{1}{1 + k}$$

- Observations.
  - gain decreases the steady-state error.
  - infinite gain is required for 'perfect' tracking.

©2002 M.C. Ramos
UP EEE Institute

EE 231

- Try something else → add an integrator.



$$\frac{N(s)}{D(s)} = \frac{1}{s} \frac{1}{s + 1} = \frac{1}{s^2 + s}$$

```
>> [n,d] = cloop(1, [1 1 0], -1)
>> step(n, d)
```


©2002 M.C. Ramos
UP EEE Institute

EE 231

- Steady-state error.

$$\frac{E}{R} = \frac{1}{1 + HG} = \frac{s^2 + s}{s^2 + s + 1}$$

- For a step input : $r(t) = u(t)$, then

$$e_{ss} = \lim_{s \to 0} s \frac{s^2 + s}{s^2 + s + 1} \frac{1}{s} = 0$$

- System type is now type 1.
  The system can track a step input with $e_{ss} = 0$.

©2002 M.C. Ramos
UP EEE Institute

EE 231

- System type 0 :

$$\lim_{s \to 0} GH = K_p < \infty$$

$$r(t) = u(t)$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s \frac{1}{1 + GH} \frac{1}{s} = \frac{1}{1 + K_p}$$

$$r(t) = tu(t)$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s \frac{1}{1 + GH} \frac{1}{s^2} = \infty$$

©2002 M.C. Ramos
UP EEE Institute

EE 231

- System type 1.

$$\lim_{s\to 0} sGH = K_v < \infty \text{ and } \lim_{s\to 0} GH = K_p = \infty$$

$$r(t) = u(t)$$
$$\Rightarrow e_{ss} = 0$$

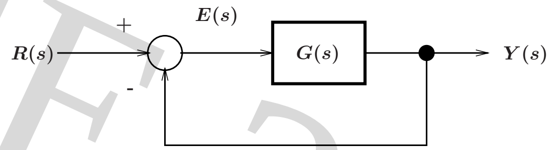
$$r(t) = tu(t)$$
$$\Rightarrow e_{ss} = \lim_{s\to 0} s \frac{1}{1 + GH}\frac{1}{s^2} = \frac{1}{K_v}$$

$$r(t) = \frac{t^2}{2}u(t)$$
$$\Rightarrow e_{ss} = \lim_{s\to 0} s \frac{1}{1 + GH}\frac{1}{s^3} = \infty$$

©2002 M.C. Ramos
UP EEE Institute

EE 231

- A detailed look at system type.



- Transfer function.

$$G(s) = k\frac{(s - z_1)\ \ldots\ (s - z_m)}{s^j(s - p_1)\ \ldots\ (s - p_n)}$$

where j is the system type.

©2002 M.C. Ramos
UP EEE Institute

EE 231

- Error equation.

$$E(s) = \frac{1}{1 + G(s)}R(s)$$

$$E(s) = \frac{1}{1 + k\dfrac{(s - z_1)\ \ldots\ (s - z_m)}{s^j(s - p_1)\ \ldots\ (s - p_n)}}R(s)$$

$$E(s) = \frac{s^j(s - p_1)\ldots(s - p_n)}{\left\{\begin{array}{l} s^j(s - p_1)\ldots(s - p_n)\ + \\ k(s - z_1)\ldots(s - z_m)\end{array}\right\}}R(s)$$

©2002 M.C. Ramos
UP EEE Institute

EE 231

- For conciseness,

$$E(s) = \frac{s^j \prod_{q=1}^{n}(s - p_q)}{s^j \prod_{q=1}^{n}(s - p_q) + k\prod_{r=1}^{m}(s - z_r)}R(s)$$

- Steady-state error.

$$e_{ss} = \lim_{s\to 0} sE(s)$$

©2002 M.C. Ramos
UP EEE Institute

- Step input, $r(t) = u(t)$.

$$e_{ss} = \lim_{s \to 0} s \frac{s^j \prod_{q=1}^{n} (s - p_q)}{s^j \prod_{q=1}^{n} (s - p_q) + k \prod_{r=1}^{m} (s - z_r)} \frac{1}{s}$$

- Error constants for a step input.
  - system type 0 : e_{ss} = finite constant.
  - system type 1 or greater : $e_{ss} = 0$.

©2002 M.C. Ramos
UP EEE Institute

- Ramp input, $r(t) = tu(t)$.

$$e_{ss} = \lim_{s \to 0} s \frac{s^j \prod_{q=1}^{n} (s - p_q)}{s^j \prod_{q=1}^{n} (s - p_q) + k \prod_{r=1}^{m} (s - z_r)} \frac{1}{s^2}$$

- Error constants for a ramp input.
  - system type 0 : $e_{ss} = \infty$.
  - system type 1 : e_{ss} = finite constant.
  - system type 2 or greater : $e_{ss} = 0$.

©2002 M.C. Ramos
UP EEE Institute

- Same procedure for parabolic input.
  - system type 0 : $e_{ss} = \infty$.
  - system type 1 : $e_{ss} = \infty$.
  - system type 2 : e_{ss} = finite constant.
  - system type 3 or greater : $e_{ss} = 0$.

- Observations.
  - system type affects steady-state error.
  - input function affects steady-state error.

©2002 M.C. Ramos
UP EEE Institute

Summary

- One can determine steady-state error constant immediately by simply looking at the system type and input function.

- One will know what system type is needed drive the steady-state error to zero for a known input.

- Add integrators to increase the system type and consequently make the steady-state error zero.

©2002 M.C. Ramos
UP EEE Institute