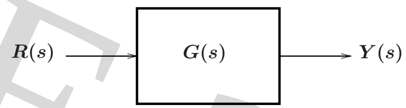
Today's EE 231 Lecture

- Some more about transfer functions.
- General control system, definitions and objectives.
- LTI response to forcing functions.
- Familiarization with Octave.

Transfer Functions

- $Y(s) = G(s)R(s)$



$$G(s) = \frac{N(s)}{D(s)} \begin{array}{l} \leftarrow \text{ numerator} \\ \leftarrow \text{ denominator} \end{array}$$

- $N(s)$ polynomial in s of order m.
  $D(s)$ polynomial in s of order n.

Transfer Functions

- Zeros of the TF : roots of $N(s)$, i.e.,
  $$s = \{z_1, z_2, \ldots, z_m\} \text{ such that } N(s) = 0.$$
- Poles of the TF : roots of $D(s)$, i.e.,
  $$s = \{p_1, p_2, \ldots, p_n\} \text{ such that } D(s) = 0.$$
- Zeros and poles affect the open-loop stability as well as the closed-loop stability of a system.

Transfer Functions

- Open-loop system.

$$G(s) = \frac{N(s)}{(s - p_1)\ldots(s - p_n)}$$

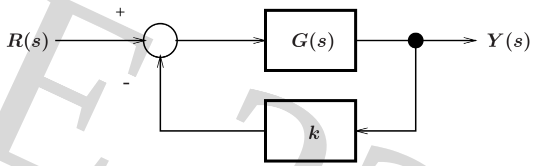
impulse response.

$$y(t) = k_1 e^{p_1 t} + \ldots + k_n e^{p_n t}$$

$\Rightarrow$ "poles affect the system response."

Transfer Functions

- Closed-loop system.

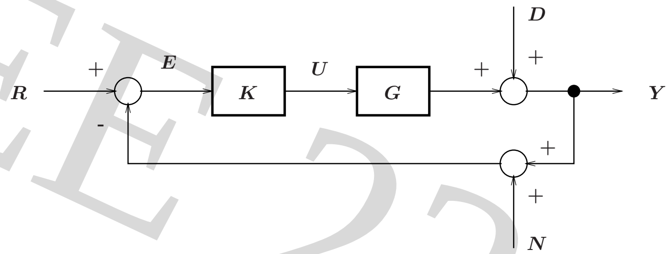


$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + kG(s)} = \frac{N(s)}{D(s) + kN(s)}$$

$\Rightarrow$ "poles and zeros affect the system response."

General Control System

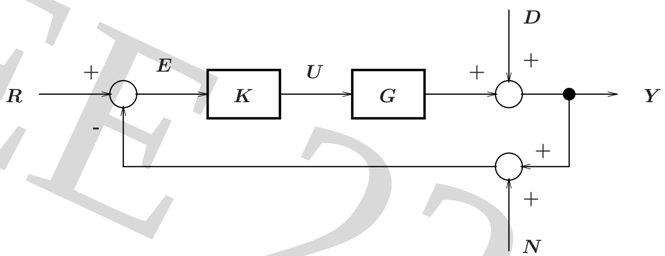
- Control system.



R : reference input.
D : disturbance.
− known / unknown
− random / deterministic
N : sensor or measurement noise.

General Control System

- Y : output.



using $Y = KGE + D$ and $E = R - (Y + N)$,

$$Y = \frac{KG}{1 + KG}R + \frac{1}{1 + KG}D - \frac{KG}{1 + KG}N$$

General Control System

- E : tracking error.
using $E = R - (Y + N)$ and $Y = KGE + D$,

$$E = \frac{1}{1 + KG}R - \frac{1}{1 + KG}D - \frac{1}{1 + KG}N$$

- U : actuator input.
since $U = KE$,

$$U = \frac{K}{1 + KG}(R - D - N)$$

Control System Objectives

- **Small error E.**
- **Small input U.**
- **Small effect of disturbance D on output Y and error E.**
- **Small effect of noise N on Y and E.**

Definitions

- **Return difference.**

$$J = 1 + KG$$

- **Sensitivity.**

$$S = \frac{1}{1 + KG} = \frac{1}{J}$$

- **Complementary sensitivity.**

$$T = 1 - S = \frac{KG}{1 + KG}$$

Control System Objectives

- **System variables.**

$$\begin{aligned} \text{output} : Y &= SD + T(R - N) \\ \text{error} : E &= S(R - D - N) \\ \text{input} : U &= KS(R - D - N) \end{aligned}$$

- **Disturbance rejection : reducing the effect of D.**
  → needs small S.

Control System Objectives

- **Good tracking : small E.**
  → needs small S.
- **Noise immunity : reducing the effect of N.**
  → needs small T, large S.
- **Bounded actuator signals : small U.**
  → needs small $KS \approx \frac{1}{G}$ for large K.

Typical Control System Design

- Large loop gain at low frequencies.
  → for tracking and rejection.

- Low loop gain at middle frequencies.
  → for stability.

- Low loop gain at high frequencies.
  → for noise immunity.

System Response

- System type and error constants.

  Can we classify a system in terms of its response to standard reference inputs (e.g. step or ramp inputs)?

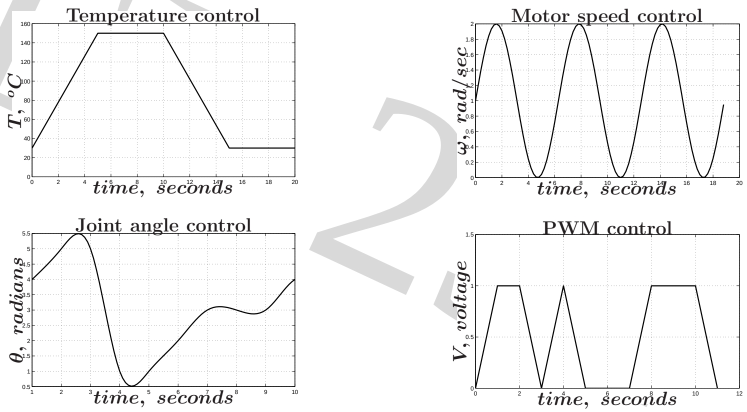
- Decompose system response.

$$y(t) = y_{transient}(t) + y_{steady-state}(t)$$

where $\lim_{t\to\infty} y_{transient}(t) = 0$.

System Response

- Examples of forcing functions.



Standard Reference Inputs

- Why use standard reference inputs?
  - take advantage of linearity and superposition.
  - simple Laplace transforms.

- Step input (e.g. setpoints).

$$u(t) \Leftrightarrow \frac{1}{s}$$

- Ramp input (e.g. gradual temp or speed changes).

$$tu(t) \Leftrightarrow \frac{1}{s^2}$$

Standard Reference Inputs

- Parabolic input (e.g. splines and robot trajectories).

$$\frac{t^2}{2}tu(t) \Leftrightarrow \frac{1}{s^3}$$

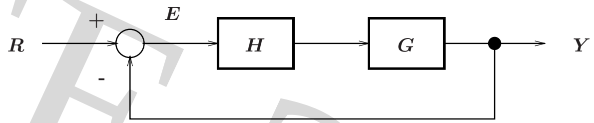
- Sinusoidal input (rotating machines).

$$cos\omega t \Leftrightarrow \frac{s}{s^2 + \omega^2}$$

$$sin\omega t \Leftrightarrow \frac{\omega}{s^2 + \omega^2}$$

Error Response

- Consider the unity gain feedback system.



$$E = \frac{1}{1 + HG}R$$

- Steady-state error.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

Error Response

- Using the final value theorem.

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s\frac{R(s)}{1 + HG(s)}$$

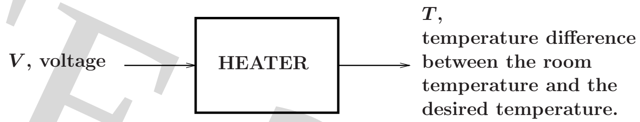
- System type.

$$HG(s) = k\frac{(s - z_1) \ldots (s - z_m)}{s^j(s - p_1) \ldots (s - p_n)}$$

definition : system type $\triangleq j$.

System Type

- Why consider system type?



simple model (first-order, type 0).

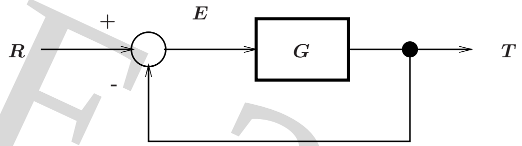
$$G(s) = \frac{1}{s + 1}$$

- Step response.

```
>> step(1, [1 1])
```

System Type

- Now add feedback.

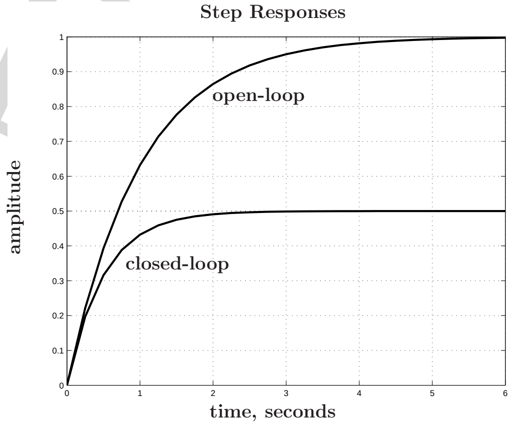


- Step response of the closed-loop system.

```
>> [n,d] = cloop(1, [1 1], -1)
>> step(n, d)
```


System Type

- Simulation results for the open-loop and closed-loop systems.



System Type

- Steady-state error.

$$\frac{E}{R} = \frac{R - T}{R} = \frac{1}{1 + G} = \frac{s + 1}{s + 2}$$

- For a step input : $r(t) = u(t)$, then

$$e_{ss} = \lim_{s \to 0} s \frac{s + 1}{s + 2} \frac{1}{s} = \frac{1}{2}$$

- For a type 0 system.
  $\Rightarrow$ non-zero tracking error in response to a step input.

Familiarization with Octave

- Polynomial in s.
  Example. Consider $a_1 s^2 + a_2 s + a_3$.

```
>> p = [a_1 a_2 a_3]
```

- Fractional polynomial in s.
  Example.

$$\frac{a_1 s^2 + a_2 s + a_3}{b_1 s^2 + b_2 s + b_3} \begin{array}{l} \Leftarrow num \\ \Leftarrow den \end{array}$$

```
>> num = [a_1 a_2 a_3]
>> den = [b_1 b_2 b_3]
```

- **Simulating time response.**
  Example. Determine the step response of

$$G(s) = \frac{10}{s^2 + 2s + 10}$$

```
>> num = 10; den = [1 2 10];
>> t = [0:0.1:10]';
>> step(num, den, t);
```

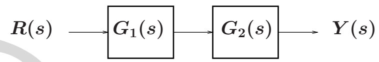
- **Determine the ramp response of $G(s)$.**

```
>> ramp = t;
>> lsim(num, den, ramp, t);
```


- **Series interconnection.**

```
>> [ns, ds] = series(n1, d1, n2, d2)
```

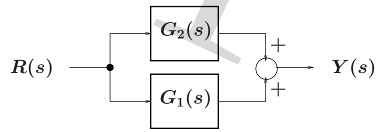
$$G_s(s) = G_1(s)G_2(s)$$



- **Parallel connection.**

```
>> [np, dp] = parallel(n1, d1, n2, d2)
```

$$G_p(s) = G_1(s) + G_2(s)$$

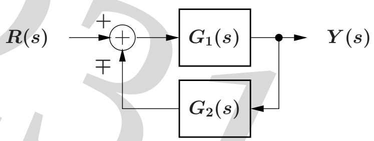


- **Feedback connection.**

```
>> [nf, df] = feedback(n1, d1, n2, d2, sign)
```

$$\texttt{sign} = \begin{cases} +1, & \text{positive feedback} \\ -1, & \text{negative feedback} \end{cases}$$

$$G_p(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)}$$



- **Exercise. Compute the step response of $G(s)$ with unity gain positive feedback. Hint.**

```
>> [nf, df] = feedback(num, den, 1, 1, +1)
```

- **Exercise. Compute the step response of $G(s)$ with unity gain negative feedback.**

- **Methods of determining response.**
  - **lsim (Matlab)**
  - **lsode (octave)**

Summary

- **Transfer functions, and poles and zeros.**

- **What do you want a control system to do?**

- **System type for closed-loop systems.**

- **Some more Octave and step response.**