EE 212 Homework 4.

1. Damped system.

   Suppose $\dot{x} = Ax$ and $\dot{z} = \sigma z + Az = (A + \sigma I)z$ where $\sigma \in R$, and $x(0) = z(0)$. How are $z(t)$ and $x(t)$ related? Find the simplest possible expression for $z(t)$ in terms of $x(t)$. Justify your answer.

   When $\sigma < 0$, some people refer to the system $\dot{z} = \sigma z + Az$ as a damped version of $\dot{x} = Ax$. Another way to think of the damped system is in terms of leaky integrators. A leaky integrator satisfies $\dot{y} - \sigma y = u$; to get the damped system, you replace every integrator in the original system with a leaky integrator.

2. Harmonic oscillator.

   The system

$$\dot{x} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} x$$

   is called a harmonic oscillator.

   (a) Find the eigenvalues, resolvent, and state transition matrix for the harmonic oscillator. Express $x(t)$ in terms of $x(0)$.

   (b) Sketch the vector field of the harmonic oscillator.

   (c) The state trajectories describe circular orbits, i. e., $||x(t)||$ is constant. Verify this fact using the solution from part (a).

   (d) You may remember that circular motion (in a plane) is characterized by the velocity vector being orthogonal to the position vector. Verify that this holds for any trajectory of the harmonic oscillator. Use only the differential equation; do not use the explicit solution you found in part (a).

3. Consider the system described by $\dot{x} = Ax$, where

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

   (a) Find e^A.

   (b) Suppose $x_1(0) = 1$ and $x_2(1) = 2$. Find x(2). (This is called a two point boundary value problem, since we are given conditions on the state at two time points instead of the usual single initial point.)