EE 212 Homework 3.

1. Find a state-space realization for the discrete-time system whose input u and output y are related by

$$y(k - 3) + 2y(k - 2) + y(k) = u(k - 2) + u(k - 1) - u(k), \ k = 0, 1, 2, \ldots$$

with $y(0) = y(1) = y(2) = 0$. Draw a block-diagram.

2. Let $A \in R^{p\times m}$ have rank r. Show that there are matrices C and B such that $A = CB$, where $C \in R^{p\times r}$ and $B \in R^{r\times m}$ are both full rank (r).

   Hint. Form C from r linearly independent columns of A.

3. Let $\mathcal{X} = \{x : R_+ \to R^n \mid x \text{ is differentiable}\}$ and $\mathcal{V} = \{x \in \mathcal{X} : \dot{x} = Ax\}$

   Thus $\mathcal{V}$ is a set of all trajectories of the autonomous linear system $\dot{x} = Ax$. Show that $\mathcal{V}$ is a vector space.

4. Proof of the Cauchy-Schwarz inequality.

   (a) Suppose $a \geq 0$, $c \geq 0$, and for all $\lambda \in R$

   $$a + 2b\lambda + c\lambda^2 \geq 0$$

   Show that $|b| \leq \sqrt{ac}$.

   (b) Given $v, w \in R^n$, explain why

   $$(v + \lambda w)^T (v + \lambda w) \geq 0$$

   for all $\lambda \in R$.

   (c) Apply (a) to the quadratic resulting when the expression in (b) is expanded, to get the Cauchy-Schwarz inequality:

   $$|v^T w| \leq \sqrt{v^T v}\sqrt{w^T w}$$

5. Which of the following is a vector space? Explain your answer. In each case, use the standard addition and scalar multiplication associated with each space. What are the dimensions of the vector spaces?

   (a) The set of integers.

   (b) The set of real valued functions on the interval $[0, T]$ which are bounded by one; that is, $\{u : [0, T] \to R \mid |u(t)| \leq 1 \text{ for } 0 \leq t \leq T\}$.

   (c) The set of $n \times n$ strictly upper triangular matrices (i.e., $a_{ij} = 0$ if $i \geq j$).

   (d) The set of $n \times n$ symmetric matrices.

6. Suppose $A \in R^{n\times n}$. The Cayley-Hamilton theorem tells us that the set of matrices $I, A, A^2, \ldots, A^n$ is dependent (in the vector space $R^{n\times n}$).

   True or False: The set $I, A, A^2, \ldots, A^{n-1}$ is independent.