EE 212 Homework 1.

1. Determine if the following systems are linear, time-invariant, and causal:

   (a)

$$y(t) = \begin{cases} t, & if\ |u(t)| \leq 1 \\ 0, & if\ |u(t)| > 1 \end{cases}$$

   (b)

$$y(t) = \begin{cases} 3u(t), & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}$$

   (c)

$$y(t) = \left(\int_0^t u(s)ds \right)^2$$

   (d)

$$y(t) = \left(\int_{t-4}^{t-1} u(s)ds \right)^2$$

2. For a linear system $\mathcal{N}$, show that the output $y = 0$ (this means that $y(t) = 0$ for all t), if the input $u = 0$.

3. Using the results of Problem 2, prove that a linear system is causal if and only if, whenever an input $u(t) = 0$ for $t \leq T$, the resultant output y satisfies $y(t) = 0$ for $t \leq T$. This is called zero-input zero-output causality.

4. Suppose a physical system is represented by the following system of equations, where $u(t)$ is the input and $y(t)$ is the output:

$$\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t-1) + |u(t)| \\ x_1(t) \end{bmatrix}, y(t) = x_1(t+1) + x_2(t) - u(t)$$

   Classify the above system according to the classification described in the notes.

5. Suppose a linear time-invariant system is connected in series with a linear time-varying system.

   (a) Is the concatenated system still linear?

   (b) If the order were reversed, would the new system have the input-output behavior as the old? Give either a proof or a counterexample.